Welcome to Algebra 2!

In the following pages, you will find review materials that will prepare you for Algebra 2. All of these topics should be review. Please take the exercises seriously, as this will allow us to hit the ground running in the fall.

If the examples preceding the practice problems are not enough of a reminder of a concept, please remember than Khan Academy, YouTube, and math.com are very useful resources.

You're also welcome to email me (beth.hill@icregina.com).

Materials Needed for Algebra 2:

- Graphing calculator* (Texas Instruments) - TI-84 Plus CE or TI-Nspire CX (not CAS). I am more familiar with the TI-84 Plus CE. We won't use the calculators for the first couple of months of class, but they are usually cheaper to purchase in the fall.
- 3-ring binder (1 or 1.5 inches) to keep notes and handouts organized
- Loose-leaf paper
- Folder with 2 pockets (if your 3-ring binder doesn't have pockets)
- Pencils (you must have a pencil for class every day)

*Graphing calculators are needed in this course and subsequent courses – even courses in college. Invest in one now, take good care of it, and use it for many years to come.

The review materials are separated into weeks. These weeks are only a suggestion. You will have the most benefit from this material if you work on it throughout the summer and do a final review of your work a week or two before school starts. You are welcome to work with your classmates but make sure that you are doing your own work. I do not tolerate copying, as I know it is very detrimental to you.

This packet must be completed by the first day of class. We will have a quiz or test over the material at the beginning of the semester that will count toward your grade.

Table of Contents:

I look forward to seeing everyone on the first day of school!

Mrs. Hill

Week 1: Algebraic Expressions & Models and Solving Linear Equations

Order of Operations	Vocabulary
Parentheses	Terms: Parts added together to make an expression
Exponents	Coefficients: The number located in front of the variable
Multiplication/Division	Constant: Number in an expression without a variable
Addition/Subtraction	

Evaluate: To **evaluate** an expression, substitute a number for each variable in the expression. Then simplify.

Remember that $(-6)^2 = -6(-6) = 36$ and $-6^2 = -(6^2) = -36$.

In #1-4, evaluate each expression for the given values of the variables.

1. $-4v + 3(w + 5v) - 5w$ for $v = -2$ and $w = 3$

2. $c(3 - a) - c^2$ for $a = 5$ and $c = -3$

3. $2(3g - 5f) + 3(g + 4f)$ for $f = -4$ and $g = 2$

4. $\frac{2(x^2-y^2)}{3}$ for $x = 6$ and $y = -4$

In #5-8, simplify by combining like terms.

5. $5x - 3x^2 + 16x^2$

6. $3y - (6y - 8)$

7. $4n - 5(n + 1)$

8. $t + \frac{t^2}{2} + t^2 + t$

Remember that to solve equations, you can add, subtract, multiply, or divide by any number or variable as long as you do the same operation to the other (entire) side.

Example:

$\frac{2}{5}(x-3)=x-2$	Original problem
$\frac{2}{5}x-\frac{6}{5}=x-2$	Distribute $\frac{2}{5}$ to each term on the left-hand side of the equation
$5\left(\frac{2}{5}x-\frac{6}{5}\right)=5(x-2)$	Multiply both sides of the equation by 5 to get rid of the fractions on the left-hand side of the equation
$2x-6=5x-10$	Distribute 5 to each term on both sides of the equation
$-6=3x-10$	Subtract $2x$ from both sides of the equation
$4=3x$	Add 10 to both sides of the equation
$x=\frac{4}{3}$	Divide both sides of the equation by 3

In #9-14, solve the equation (show each step in your solution process). Check your answer (plug your answer into the original equation and make sure the equation is true).

9. $9(z+2)=12z$

10. $8y+7=6y+11$

11. $5w-8+9w=16-18w$

12. $3(x+2)=2(x+11)$

13. $\frac{1}{3}(y-2)=y+6$

14. $4-\frac{2}{3}x=-5$

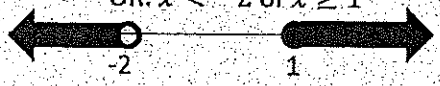
Week 2: Solving Linear Inequalities & Finding the Slope of a Line

*If you multiply or divide each side of an inequality by a negative number, switch the inequality symbol.
*Closed dot represents $\leq$ and $\geq$. This means the value is included in the solution.
*Open dot represents $<$ and $>$. This means the value is NOT included in the solution.

Compound Inequalities: Two simple inequalities joined by the word "and" or "or."

AND: $-2 \leq x < 1$

OR: $x < -2$ or $x \geq 1$



Example:

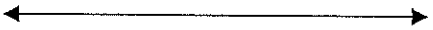
$7 < -6x + 1 \leq 13$	Original problem
$6 < -6x \leq 12$	Subtract 1 from each side of the equation
$-1 > x \geq -2$	Divide both sides of the equation by -6 (which causes you to switch $<$ to $>$ and switch $\leq$ to $\geq$)
$-2 \leq x < -1$	Rewrite the inequality with the smaller number on the left
	Graph

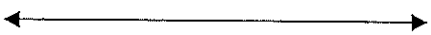
For #1-3, write the inequality that represents the sentence. Do not solve.

1. Six less than a number is at least -4.
2. The product of a number and 7 is at most -11.
3. Six more than a quotient of a number and 3 is greater than 13.

For #4-5, solve each inequality. Graph the solution.

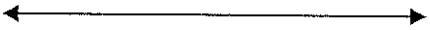
4. $3a - 10 > 5$
5. $-5(n + 2) + 6 \leq 16$





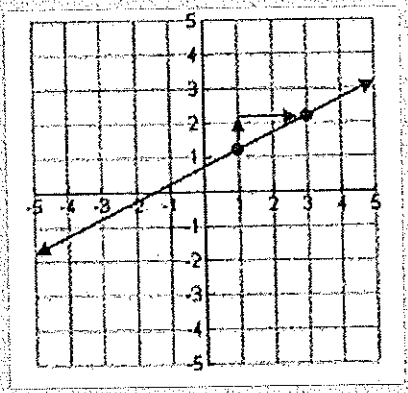
For #6-7, solve each compound inequality. Graph the solution.

6. $-2 \leq 3x - 8 < 10$
7. $2x + 3 < 12$ or $4x - 7 \geq 21$

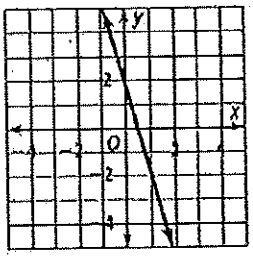


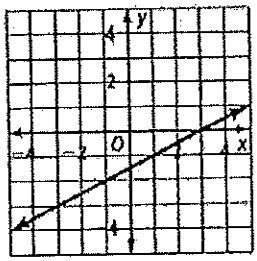
Finding Slope (m)

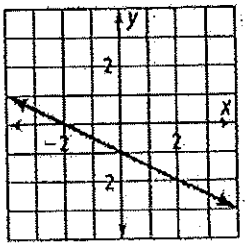
$$m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

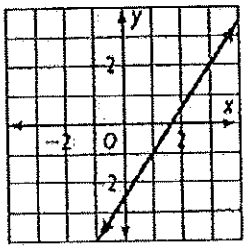
Finding Slope From A Graph	Finding Slope From 2 Points
 $\Delta y = 1$ (rise) $\Delta x = 2$ (run) $m = \frac{1}{2}$	Example: Find the slope of the line between $(-2, 7)$ and $(3, -1)$. $m = \frac{-1-7}{3-(-2)} = \frac{-8}{5}$ $m = -\frac{8}{5}$

For #8-15, find the slope of the following lines (pick 2 points on each line) or points.

8. 

9. 

10. 

11. 

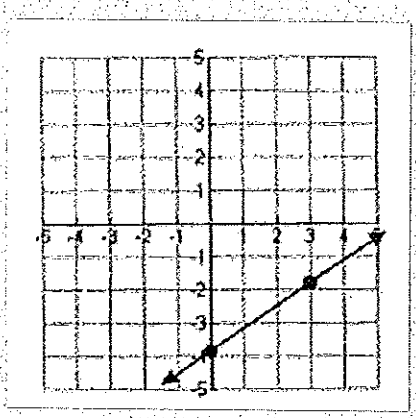
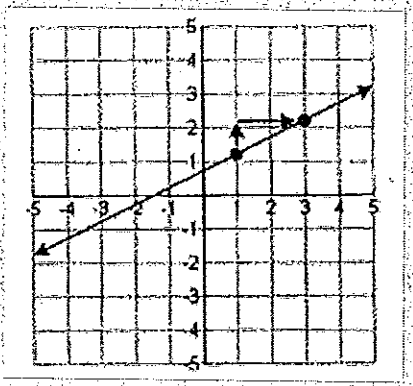
12. $(8, 10)$ and $(-7, 12)$

13. $(-18, -2)$ and $(8, -2)$

14. $(5, -2)$ and $(1, 6)$

15. $(3, 7)$ and $(3, -5)$

Week 3: Graphing Linear Equations & Linear Inequalities

Slope-Intercept Form	**Point-Slope Form**
$y = mx + b$ m is the slope of the line b is the y-intercept of the line	$y - y_1 = m(x - x_1)$ m is the slope of the line (x_1, y_1) is a point on the line
Example: Graph $y = \frac{2}{3}x - 4$ y-intercept is -4 or $(0, -4)$ Slope is $\frac{2}{3} \rightarrow$ up 2, right 3 	Graph the point and then use the slope to graph more points (using rise over run). Example: Graph $y - 1 = \frac{1}{2}(x - 1)$ Point: $(1, 1)$ Slope: $\frac{1}{2} \rightarrow$ up 1, right 2 

Identifying the x- and y-intercepts

The x-intercept is the point where the graph crosses the x-axis (where $y = 0$). Substitute $y = 0$ to find the x-value of this point.
The y-intercept is the point where the graph crosses the y-axis (where $x = 0$). Substitute $x = 0$ to find the y-value of this point.

Example:
Find the x- and y-intercepts for $-3 = 3(x + 1)$.

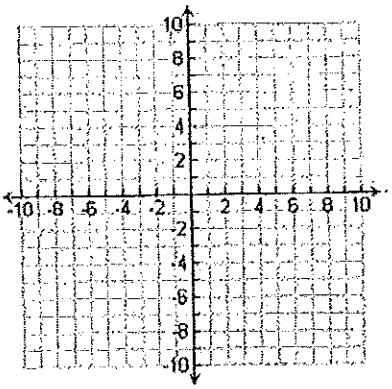
x-intercept	y-intercept
Substitute in $y = 0$: $0 - 3 = 3(x + 1)$ Solve: $-3 = 3(x + 1)$ Distribute 3 through to $x + 1$: $-3 = 3x + 3$ Subtract 3 from both sides: $-6 = 3x$ Divide both sides by 3: $x = -2$ x-intercept: $(-2, 0)$	Substitute in $x = 0$: $y - 3 = 3(0 + 1)$ Solve: $y - 3 = 3(1)$ Simplify on the right-hand side: $y - 3 = 3$ Add 3 to both sides: $y = 6$ y-intercept: $(0, 6)$

For #1-6, find the requested information and graph the equation.

1. $y = -2x - 1$

Slope: _____

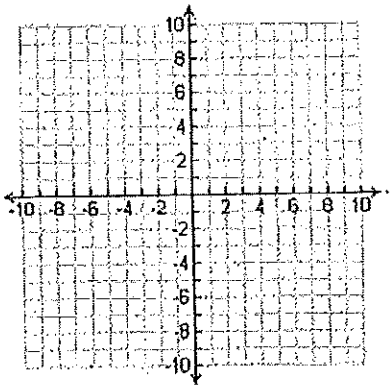
y-intercept: (____, ____)



2. $y = 3x - 2$

Slope: _____

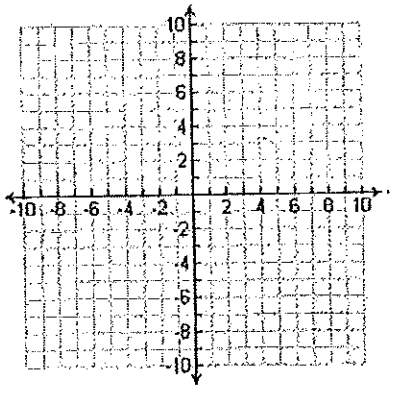
y-intercept: (____, ____)



3. $y = -\frac{2}{3}x + 4$

Slope: _____

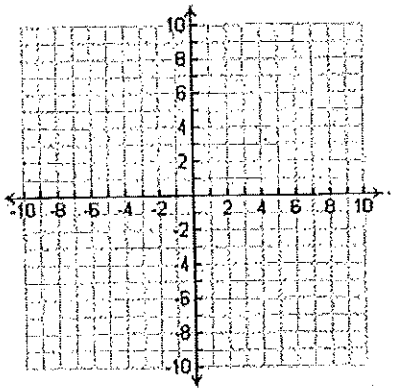
y-intercept: (____, ____)



4. $y - 1 = \frac{2}{3}(x + 4)$

Slope: _____

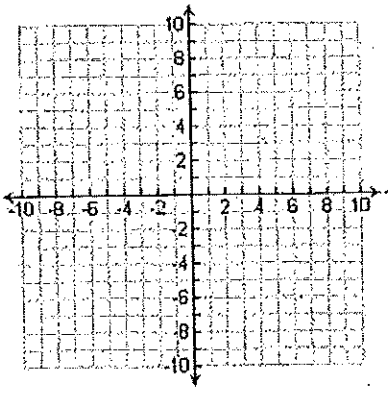
Point: (____, ____)



5. $y + 2 = -2(x - 1)$

Slope: _____

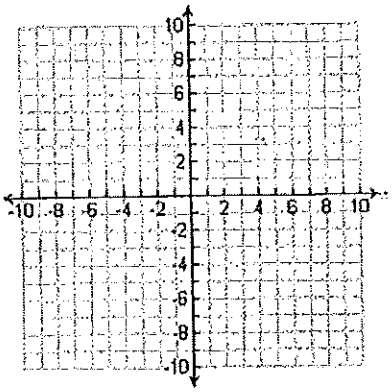
Point: (____, ____)



6. $y + 3 = \frac{5}{3}(x + 8)$

Slope: _____

Point: (____, ____)



For #7-9, find the intercepts of the equation. Show your work (substitute 0 in for x or y and solve for the other variable).

7. $x + y = 7$

x-intercept : (____, ____)

y-intercept : (____, ____)

8. $2x - y = 8$

x-intercept : (____, ____)

y-intercept : (____, ____)

9. $3x - 4y = 10$

x-intercept : (____, ____)

y-intercept : (____, ____)

Graphing Linear Inequalities

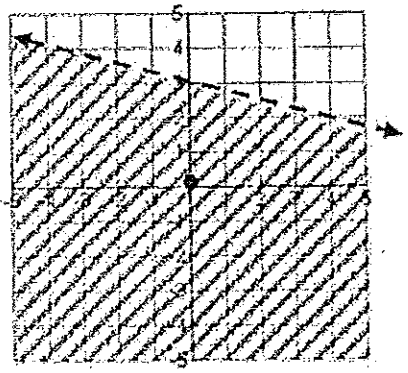
1. Graph the line the same way you would any other linear equation.
2. Remember $<$ or $>$ represents a dashed line and $\leq$ or $\geq$ represents a solid line.
3. Choose a test point on the graph to see if it satisfies the inequality. If it does, shade to cover the test point as it is a solution. If it is not, shade away from it.

Example: Graph $y < -\frac{1}{4}x + 3$

1. Graph $y = -\frac{1}{4}x + 3$
2. Graph has a dashed line.
3. Test Point: (0,0): $0 < -\frac{1}{4}(0) + 3$

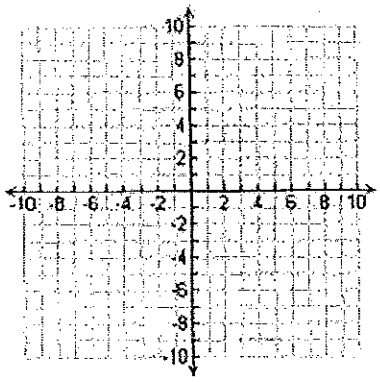
   $0 < 3 \rightarrow$ TRUE

   Shade to cover (0,0).

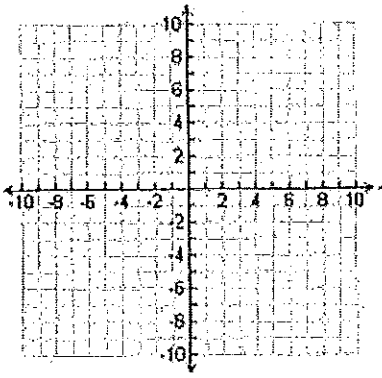


Graph the linear inequalities.

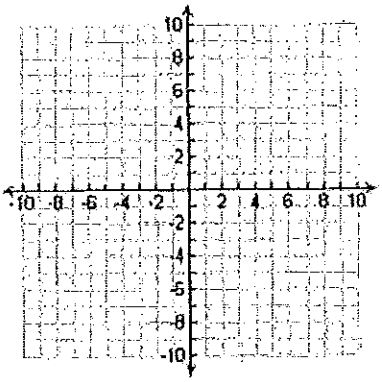
1. $3x + y \geq 6$



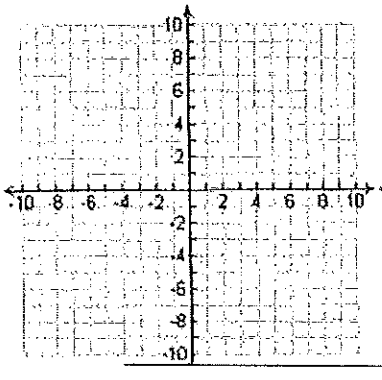
2. $x + y < -2$



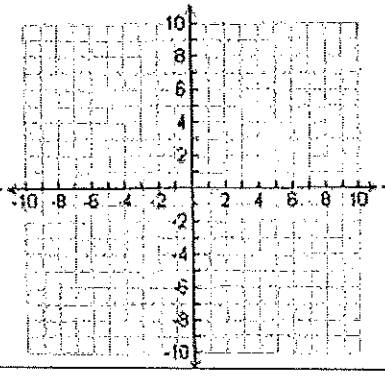
3. $x + 4y \leq 8$



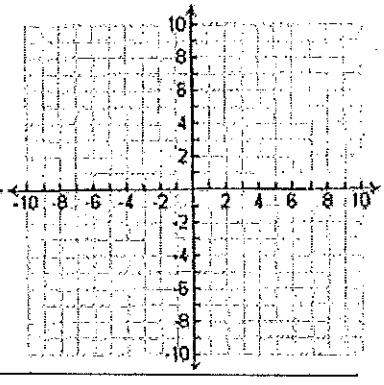
4. $y \leq \frac{3}{4}x + 1$



5. $y < -x + 4$



6. $y \geq -\frac{2}{5}x - 2$



Week 4: Solving Systems of Equations

from Prentice Hall Algebra 2 • Teaching Resources

Substitution Method:

Step 1: Solve one equation for one of the variables.

Step 2: Substitute the expression for this first variable into the other equation. Solve for the second variable.

Step 3: Substitute the second variable's value into either equation. Solve for the first variable.

Step 4: Check the solution in the other original equation.

Example:

What is the solution of the system of equations shown? $\begin{cases} 4x + 3y = 10 \\ x + 2y = 10 \end{cases}$

Step 1:	$x = -2y + 10$	Solve one equation for x.
Step 2:	$4(-2y + 10) + 3y = 10$ $-8y + 40 + 3y = 10$ $-5y + 40 = 10$ $-5y = -30$ $y = 6$	Substitute the expression for x into the top equation. Distribute. Collect like terms ($-8y + 3y = -5y$). Collect like terms (subtract 40 from both sides). Divide both sides of the equation by -5.
Step 3:	$x + 2(6) = 10$ $x + 12 = 10$ $x = -2$	Substitute the y-value into the bottom equation. Simplify 2(6). Subtract 12 from both sides of the equation.
Step 4:	$4(-2) + 3(6) = 10$ $-8 + 18 = 10$ $10 = 10$	Check the solution in the top equation. Simplify. $x = -2, y = 6$ makes the equation true, so the solution works.

The solution is $(-2, 6)$.

For #1-4, solve each system by substitution.

1. $\begin{cases} x - 3y = 2 \\ -x + 2y = 5 \end{cases}$

2. $\begin{cases} a + 3b = 4 \\ a = -2 \end{cases}$

3. $\begin{cases} -2m + n = 6 \\ -7m + 6n = 1 \end{cases}$

4. $\begin{cases} 7x - 3y = -1 \\ x + 2y = 12 \end{cases}$

Elimination Method:

Step 1: Arrange the equations with like terms in columns. Circle the like terms for which you want to obtain coefficients that are opposites.

Step 2: Multiply each term of one or both equations by an appropriate number.

Step 3: Add the equations.

Step 4: Solve for the remaining variable.

Step 5: Substitute the value obtained in Step 4 into either of the original equations, and solve for the other variable.

Step 6: Check the solution in the other original equation.

Example:

What is the solution of the system of equations shown? $\begin{cases} 2x + 5y = 11 \\ 3x - 2y = -12 \end{cases}$

Step 1:	$2x + 5y = 11$ $3x - 2y = -12$	Circle the terms that you want to make opposite.
Step 2:	$6x + 15y = 33$ $-6x + 4y = 24$	Multiply each term of the first equation by 3. Multiply each term of the second equation by -2.
Step 3:	$19y = 57$	Add the equations.
Step 4:	$y = 3$	Solve for the remaining variable.
Step 5:	$3x - 2(3) = -12$ $3x - 6 = -12$ $3x = -6$ $x = -2$	Substitute 3 for y to solve for x. Multiply 2(3). Add 6 to both sides of the equation. Divide both sides of the equation by 3.
Step 6:	$2(-2) + 5(3) = 11$ $-4 + 15 = 11$ $11 = 11$	Check using the other equation.

The solution is $(-2, 3)$.

For #5-8, solve each system by elimination.

5. $\begin{cases} 3x + 2y = -17 \\ x - 3y = 9 \end{cases}$

6. $\begin{cases} 5f + 4m = 6 \\ -2f - 3m = -1 \end{cases}$

7. $\begin{cases} 3x - 2y = 5 \\ -6x + 4y = 7 \end{cases}$

8. $\begin{cases} -2x - 4y = 2 \\ 10x + 20y = -10 \end{cases}$

Week 5: Factoring & Properties of Exponents

FOIL (first, outer, inner, last): A method of multiplying out 2 binomial expressions.
$(x+3)(x-5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$

Factoring a polynomial expression: "undoing" FOIL

Example 1: $x^2 - 9x + 20$

Find two binomial expressions with first terms that multiply to equal x^2 and last terms that equal 20.

If the constant term (20) is positive, then the signs in the binomial expressions will both be the same. If the constant term was negative, the signs in the binomial expressions would be different.

Try different combinations until you find one whose outer and inner terms equal the middle term $(-9x)$.

$(x-4)(x-5)$ since $x \cdot x = x^2$, $-4(-5) = 20$, and $x(-5) + (-4)(x) = -5x - 4x = -9x$

Example 2: $6x^2 + 5x - 21$

$(3x+7)(2x-3)$ since $2x \cdot 3x = 6x^2$, $-7(3) = -21$, and $3x(-3) + 7(2x) = -9x + 14x = 5x$

Difference of Squares: An expression that is the difference of two squared expressions. To factor a difference of squares, find an expression that is multiplied by itself to obtain the first squared expression and put it in the front of each binomial expression. Then, find an expression that is multiplied by itself to obtain second squared expression and put it in the back of each binomial expression. One binomial expression will use a plus sign and the other will use a minus sign (so that the outer and inner terms will cancel out).

Example 3: $25x^2 - 49 = (5x+7)(5x-7)$

Example 4: $4x^2 - 9y^2 = (2x+3y)(2x-3y)$

For #1-12, factor each expression. If your first guess doesn't work, leave it there and try again.

1. $x^2 + 10x + 24$
2. $x^2 - 5x - 14$
3. $x^2 + 6x - 27$
4. $x^2 - 6x + 9$
5. $2x^2 + 11x + 12$
6. $5x^2 + 34x - 7$
7. $6x^2 - 17x + 5$
8. $8x^2 + 2x - 15$
9. $21x^2 + 17x + 2$
10. $x^2 - 16$
11. $9x^2 - 25$
12. $36x^2 - 81$

Properties of Exponents:

$a^0 = 1$ $\quad a^m \cdot a^n = a^{m+n}$ $\quad a^{-n} = \frac{1}{a^n}$

$\frac{1}{a^{-n}} = a^n$ $\quad \frac{a^m}{a^n} = a^{m-n}$ $\quad (a^m)^n = a^{m \cdot n}$

$(ab)^n = a^n b^n$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

For #13-24, simplify each exponential expression. Your final answer should have only positive exponents.

13. $(4a^3)(5a^6)$

14. $(-2x^5)(-4x^{-5})$

15. $(3x^3y^2)^2$

16. $(4x^{-3}y^4)^2$

17. $(-3m^3n^2)(2mn)$

18. $(x^{-3})^{-2}$

19. $(x^5y)^3(xy)$

20. $\frac{8a^5}{2a^2}$

21. $\frac{(6x^4)^0}{3xy^3}$

22. $\frac{12x^7y^4}{3x^{-3}}$

23. $\left(\frac{2x^5}{3}\right)^2$

24. $\frac{x^3y}{9} \cdot \frac{18x}{y}$

Week 6: The Pythagorean Theorem & Special Triangles

Pythagorean Theorem

$a^2 + b^2 = c^2$

Common Pythagorean Triples

3-4-5, 5-12-13, 7-24-25, 8-15-17, 20-21-29

Simplifying Radicals:

Break radicals into smaller numbers so that you can combine them and simplify.

$\sqrt{18} \cdot \sqrt{14} = (\sqrt{9} \cdot \sqrt{2}) \cdot (\sqrt{2} \cdot \sqrt{7}) = \sqrt{9} \cdot (\sqrt{2} \cdot \sqrt{2}) \cdot \sqrt{7} = 3 \cdot 2 \cdot \sqrt{7} = 6\sqrt{7}$

For #1-6, find the missing side length. Simplify any radical answers.

1. $a = 6, c = 10$

2. $a = 20, b = 21$

3. $b = 8, c = 17$

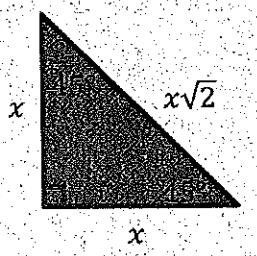
4. $a = 2, b = 3$

5. $a = 3, b = 6$

6. $a = 5, c = 7$

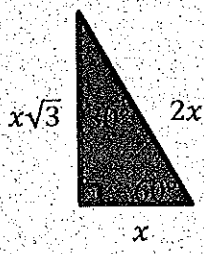
Special Right Triangles

$45° - 45° - 90°$ triangle relationship



- To get from a leg to the hypotenuse, multiply by $\sqrt{2}$.
- To get from the hypotenuse to a leg, divide by $\sqrt{2}$.

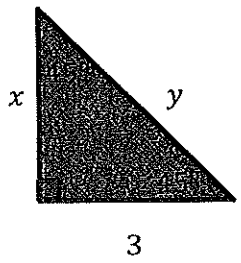
$30° - 60° - 90°$ triangle relationship



- To get from the short leg to the long leg, multiply by $\sqrt{3}$.
- To get from the short leg to the hypotenuse, multiply by 2.
- To get from the long leg to the short leg, divide by $\sqrt{3}$.
- To get from the hypotenuse to the short leg, divide by 2.

For #7-12, find the missing side lengths. Simplify any radical answers (no radicals in the denominator of an answer).

7.



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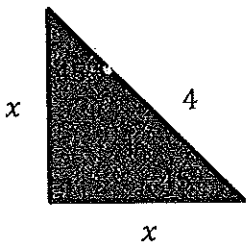

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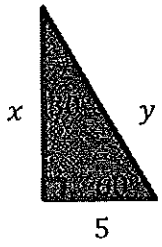

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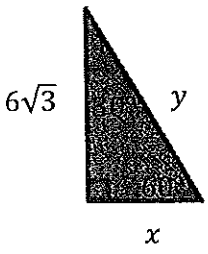

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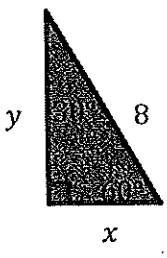

12.