

Welcome to Calculus! I'm so glad that you'll be a student in my classroom again.

In the following pages, you will find review materials that will prepare you for Calculus. All of these topics should be review. Please take the exercises seriously, as this will allow us to hit the ground running in the fall.

If the examples preceding the practice problems are not enough of a reminder of a concept, please remember that Khan Academy, YouTube, and math.com are very useful resources.

You're also welcome to email me (beth.hill@icregina.com).

Materials Needed for Calculus:

- Graphing calculator (Texas Instruments) - TI-84 Plus CE or TI-Nspire CX (not CAS). I am more familiar with the TI-84 Plus CE.
- Two 3-ring binders (1 or 1.5 inches) to keep notes and handouts organized (I think it's easier to separate notes and handouts; I'm not sure they'd all fit in one binder)
- Loose-leaf paper
- Folder with 2 pockets (if your 3-ring binder doesn't have pockets)
- Pencils (you must have a pencil for class every day, and I won't be able to provide pencils due to COVID)

You will have the most benefit from this material if you work on it throughout the summer and do a final review of your work a week or two before school starts. You are welcome to work with your classmates but make sure that you are doing your own work. I do not tolerate copying, as I know it is very detrimental to you.

This packet must be completed by the first day of class.

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I look forward to seeing everyone on the first day of school!

Mrs. Hill

*Most problems from the review packet are from mastermathmentor.com and are used with permission.

Week 1: Equations and Their Graphs & Using a Graphing Calculator

For #1-4, solve each equation without a calculator. Show all work.

1. $3x + 17 = 2$

2. $9x - 13 = 6x + 8$

3. $|x + 4| = 2$

4. $|2x + 5| = 3$

5. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

a. $m = -7, (-3, -7)$

b. $m = -\frac{1}{2}, (2, -8)$

c. $m = \frac{2}{3}, (-6, \frac{1}{3})$

6. Find the equation of the line in slope-intercept form, passing through $(-7, 1)$ and $(3, -4)$.

7. Write equations of the line through $(-6, 2)$ a) parallel and b) normal (perpendicular) to $5x + 2y = 7$.

8. Find the equation of the line in point-slope form containing $(4, -2)$ and parallel to the line containing $(-1, 4)$ and $(2, 3)$.

9. If $f(x) = 4x - x^2$, find:

a. $f(4) - f(-4)$

b. $\frac{f(x+h) - f(x-h)}{2h}$

10. If $V(r) = \frac{4}{3}\pi r^3$, find:

a. $V\left(\frac{3}{4}\right)$

b. $V(r+1) - V(r-1)$

For #11-13, show work to determine if the following functions are even, odd, or neither.

11. $f(x) = \sqrt{x^2 + 1}$

12. $f(x) = 2x^2 - 4x$

13. $f(x) = 8x - \frac{1}{8x}$

For #14-20, find any vertical and horizontal asymptotes, and if present, the location of holes, for each graph.

14. $y = \frac{2x+16}{x+8}$

15. $y = \frac{x}{x^2-25}$

16. $y = \frac{2x^2+6x}{x^2+5x+6}$

17. $y = \frac{5x+1}{x^2-x-1}$

18. $y = \frac{1-x-5x^2}{x^2+x+1}$

19. $y = \frac{10x+20}{x^3-2x^2-4x+8}$

20. $y = \frac{1}{x} - \frac{x}{x+2}$ (hint: express with a common denominator)

For #21-27, solve these equations or inequalities using a graphing calculator

21. $3x^3 - x - 5 = 0$

23. $x^3 - 5x^2 + 4x - 1 = 0$

24. $2x^2 - 1 = 2^x$

25. $2 \ln(x + 1) = 5 \cos x$ on $[0, 2\pi)$

26. $x^4 - 9x^2 - 3x - 15 < 0$

27. $\frac{x^2-4x-4}{x^2+1} > 0$ on $[0, 8]$

28. $x \sin x^2 > 0$ on $[0, 8]$

29. $\cos^{-1} x > x^2$ on $[-1, 1]$

Week 2: Function Composition, Piecewise Functions, Inverse Functions, Polynomial Long Division, and Synthetic Division

For #1-6, use $f(x) = x - 5$ and $g(x) = x^2 - 2$ to find the given composed functions. Show all work.

1. $f(g(-4))$ 2. $g(g(0))$ 3. $f(g(x))$

4. $g(f(x))$ 5. $f(f(x))$ 6. $g(f(x - 1))$

7. If $f(x) = \begin{cases} \sqrt{x+2} - 2, & x \geq 2 \\ x^2 - 1, & 0 \leq x < 2, \text{ find:} \\ -x, & x < 0 \end{cases}$

a. $f(0) - f(2)$ b. $\sqrt{5 - f(-4)}$ c. $f(f(3))$

For #8-13, find the inverse for the given function.

8. $2x - 6y = 1$ 9. $y = ax + b$ 10. $y = 9 - x^2$

11. $y = \sqrt{1 - x^3}$ 12. $y = \frac{9}{x}$ 13. $y = \frac{2x+1}{3-2x}$

14. Use polynomial long division to solve $y = \frac{x^4+3x^2-5x+9}{x^2-2x+2}$.

For #15-17, use synthetic division to perform the indicated division.

15. $y = \frac{x^3-27}{x-3}$

16. $y = \frac{x^3+2x^2-3x+8}{x+2}$

17. $y = \frac{2x^4-6x^3-4x-1}{x-1}$

Week 3: Factoring and Radical/Rational Expressions

For #1-8, completely factor the following expressions.

1. $x^3 - 25x$

2. $3x^3 - 5x^2 + 2x$

3. $3x^8 - 3$

4. $9a^4 - a^2b^2$

5. $16x^4 - 24x^2y + 9y^2$

6. $4x^4 + 7x^2 - 36$

7. $2x^3 + 6x^2 - 5x - 15$

8. $x^3 - xy^2 + x^2y - y^3$

For #9-13, solve the following equations.

9. $x^2 + 7x - 18 = 0$

10. $20x^2 - 56x + 15 = 0$

11. $x + \frac{1}{x} = \frac{17}{4}$

12. $\frac{1}{x-3} - \frac{1}{x+3} = \frac{10}{x^2-9}$

13. $\frac{2x-1}{x-1} - \frac{3x}{2x+1} = \frac{x^2+11}{2x^2-x-1}$

For #14-19, simplify the following expressions and write with positive exponents.

14. $(-12x^5)^{-2}$

15. $(4x^{-1})^{-1}$

16. $\left(\frac{-4}{x^4}\right)^{-3}$

17. $\left(\frac{5x^3}{y^2}\right)^{-3}$

18. $(121x^8)^{\frac{1}{2}}$

19. $(8x^2)^{-\frac{4}{3}}$

For #20-23, factor the following expressions completely.

20. $3x^2 - 2x + 9 - \frac{6}{x}$

21. $5x^2 + 14 - 3x^{-2}$

22. $x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}}$

23. $4x^{\frac{1}{2}} - 12x^{-\frac{1}{2}}$

Week 4: Exponents and Logarithms

For #1-6, solve the exponential equations using properties of exponents (no calculators).

1. $5^{4x+7} = 25^{6x-2}$

2. $283^{7x-5} = 1$

3. $2^{3x-4} = \left(\frac{1}{16}\right)^3$

4. $\frac{4^{5x+1}}{16^{2x-6}} = 8^5$

5. $\left(\frac{1}{49}\right)^{x-3} = 7^{\frac{1}{5}}$

6. $\left(\frac{1}{27}\right)^{3x+2} = \left(\frac{1}{9}\right)^{2x-5}$

For #7-9, evaluate the following logarithmic expressions (no calculators).

7. $\log_8 4$

8. $\log_3(\sqrt{3})^5$

9. $\ln\left(\frac{1}{\sqrt[3]{e^2}}\right)$

For #10-13, solve the following logarithmic expressions (no calculators).

10. $\log_5(3x - 8) = 2$

11. $\log_9(x^2 - x + 3) = \frac{1}{2}$

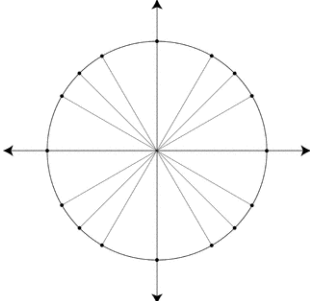
12. $\log_2(x - 1) + \log_2(x + 3) = 5$

13. $\ln x^3 - \ln x^2 = \frac{1}{2}$

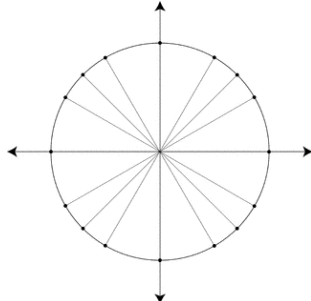
Week 5: Trigonometry

For #1-6, find the exact value of the trigonometric expression (no calculators). Mark the angle on the unit circle provided.

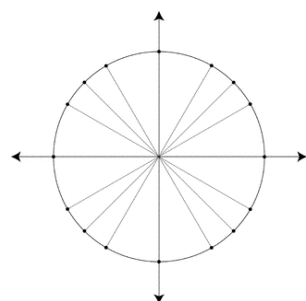
1. $\cos \frac{3\pi}{4}$



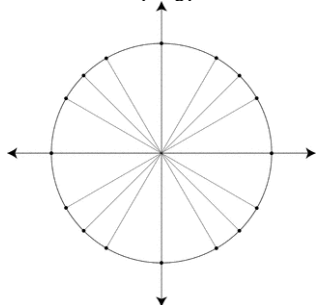
2. $\sin \pi$



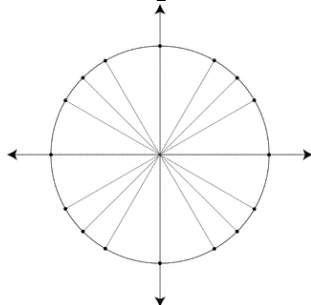
3. $\cos \frac{5\pi}{3}$



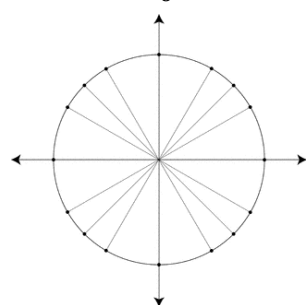
4. $\sin \left(-\frac{\pi}{3}\right)$



5. $\tan \frac{3\pi}{2}$

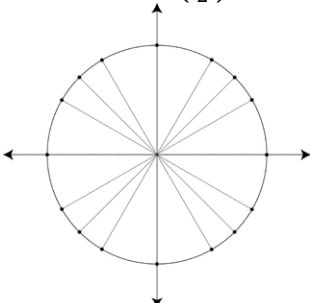


6. $\tan \frac{7\pi}{6}$

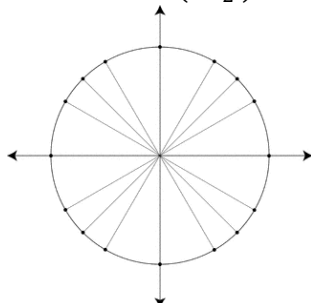


For #7-12, find the exact value of the inverse trigonometric expression in radians (no calculators). Mark the angle on the unit circle provided. Note: $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ for $\sin^{-1} y$ and $\tan^{-1} y$, and $0 \leq \theta \leq \pi$ for $\cos^{-1} y$.

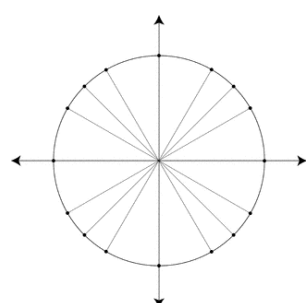
7. $\cos^{-1} \left(\frac{\sqrt{3}}{2}\right)$



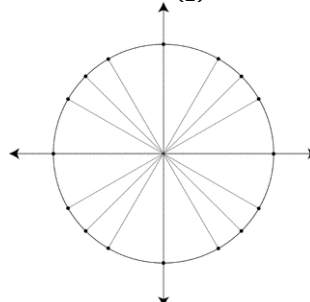
8. $\sin^{-1} \left(-\frac{\sqrt{2}}{2}\right)$



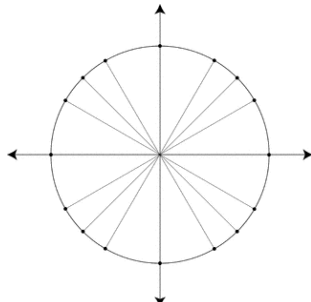
9. $\cos^{-1}(-1)$



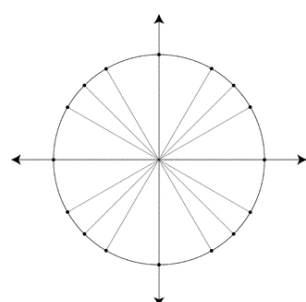
10. $\sin^{-1} \left(\frac{1}{2}\right)$



11. $\tan^{-1}(1)$



12. $\tan^{-1}(-\sqrt{3})$



For #13-16, verify each trigonometric identity.

13. $(1 + \sin x)(1 - \sin x) = \cos^2 x$

14. $\sec^2 x + 3 = \tan^2 x + 1$

15. $\csc(2x) = \frac{\csc x}{2 \cos x}$

16. $\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$

For #17-20, solve the trigonometric equation (no calculators) for $[0, 2\pi]$.

17. $\sin x = \cos x$

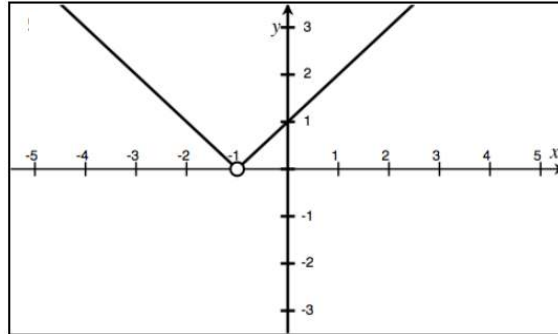
18. $\sin^2 x = \sin x$

19. $\sin^2 x = 3 \cos^2 x$

20. $2\cos^2 x + \sin x - 1 = 0$

Week 6: Limits

For #1-8, find the limit using the given graph of $f(x)$.

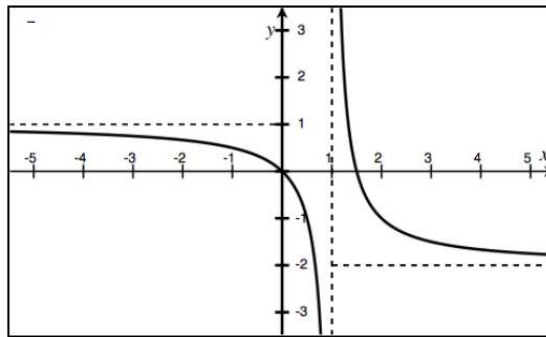


1. $\lim_{x \rightarrow -3} f(x) =$

2. $\lim_{x \rightarrow -1} f(x) =$

3. $\lim_{x \rightarrow -\infty} f(x) =$

4. $\lim_{x \rightarrow \infty} f(x) =$



5. $\lim_{x \rightarrow 0} f(x) =$

6. $\lim_{x \rightarrow 1} f(x) =$

7. $\lim_{x \rightarrow -\infty} f(x) =$

8. $\lim_{x \rightarrow \infty} f(x) =$

For #9-14, find the limit (no calculators).

9. $\lim_{x \rightarrow 7} -3 =$

10. $\lim_{x \rightarrow -2} 5x^2 - 3x + 6 =$

11. $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x - 3} =$

12. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 3x - 10} =$

13. $\lim_{t \rightarrow -3} \frac{\frac{1}{3} + \frac{1}{t}}{3 + t} =$

14. $\lim_{x \rightarrow 0} \frac{\sqrt{x+25} - 5}{x} =$