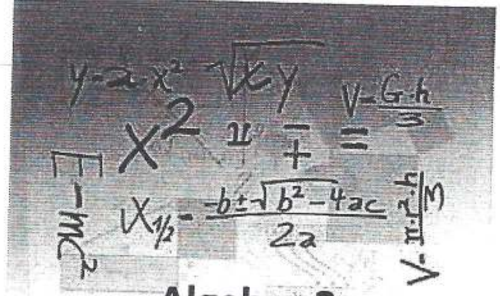
Regina Summer Math Review

For students who will be taking

Algebra 2

Completed review packet due the first day of classes



Algebra 2
Summer Review Packet

Welcome to Algebra 2!

In the following pages, you will find review materials that will prepare you for Algebra 2. Please take the exercises seriously, as this will allow us to hit the ground running in the fall. If the examples preceding the practice problems are not enough of a reminder of a concept, please remember than Khan Academy, YouTube, and math.com are very useful resources.

The review materials are separated into weeks. These weeks are only a suggestion. You will have the most benefit from this material if you work on it throughout the summer and do a final review of your work a week or two before school starts.

*****This packet must be completed by the first day of class.*****

Table of Contents:

Materials Needed for Algebra 2:

Graphing calculator (Texas Instruments) - TI-83 or TI-84 (or their families) or TI-Nspire CX (not CAS)

3-ring binder (1 or 1.5 inches)

Loose-leaf paper

Pencils

Graphing calculators are needed in this course and subsequent courses – even courses in college. Invest in one now, take good care of it, and use it or many years to come.

Week 1: Algebraic Expressions and Models

Order of Operations	Vocabulary
Parentheses Exponents Multiplication/Division Addition/Subtraction	Terms: Parts added together to make an expression. Coefficients: The number located in front of the variable. Constant: Numbers in an expression without a variable.

Write an algebraic expression that models each word phrase.

1. seven less than the number t — 1.________________
2. the sum of 11 and the product of 2 and a number r — 2.________________

Write an algebraic expression that models each situation.

3. Arin has \$520 and is earning \$75 each week babysitting. — 1.________________
4. You have 50 boxes of raisins and are eating 12 boxes each month. — 2.________________

Evaluate each expression for the given values of the variables.

5. $-4v + 3(w + 2v) - 5w$ $\quad v = -2 \quad w = 4$ — 5.________________
6. $c(3 - a) - c^2$ $\quad a = 4 \quad c = -1$ — 6.________________
7. $2(3g - 5f) + 3(g + 4f)$ $\quad g = 3 \quad f = -5$ — 7.________________

Simplify by combining like terms.

8. $5x - 3x^2 + 16x^2$
9. $\frac{2(a-b)}{9} + \frac{4}{9}b$
10. $t + \frac{t^2}{2} + t^2 + t$
11. $4a - 5(a + 1)$

8.________________

9.________________

10.________________

11.________________

Identify the following components from the expression $5x^7 - 8x + 47$

12. The number of terms — 12.________________
13. Leading coefficient — 13.________________
14. Constant Term — 14.________________

Week 2: Solving Linear Equations

Remember to solve equations, you can add, subtract, multiply or divide by any number or variable as long as you do the same operation to the other (entire) side.

Example:

$\frac{2}{5}(x-3) = x-2$	Original Problem
$\frac{2}{5}x - \frac{6}{5} = x-2$	Distribute the $\frac{2}{5}$ to each term on the left side of the =
$5\left(\frac{2}{5}x - \frac{6}{5}\right) = 5(x-2)$	Multiply both sides of the equation by 5 to get rid of the fractions on the left
$2x - 6 = 5x - 10$ $-5x \quad -5x$	Subtract $5x$ on each side
$-3x - 6 = -10$ $+6 \quad +6$	Add 6 on each side
$\frac{-3x}{-3} = \frac{-4}{-3}$	Divide both sides by -3
$x = \frac{4}{3}$	Solve for x.

Check your solution by plugging the value into the original equation.

Solve each equation.

1. $9(z-3) = 12z$ 1.________
2. $7y + 5 = 6y + 11$ 2.________
3. $5w + 8 - 12w = 16 - 15w$ 3.________
4. $3(x+1) = 2(x+11)$ 4.________
5. $\frac{1}{3}(y-2) = y + 4$ 5.________
6. $4 - \frac{2}{3}x = -7$ 6.________

Write an equation and solve each problem.

7. Two brothers are saving money to buy tickets to a concert. Their combined savings is $55. One brother has $15 more than the other. How much has each saved?
8. What three consecutive numbers have a sum of 126?
9. Two trains left a station at the same time. One traveled north at a certain speed and the other traveled south at twice that speed. After 4 hours, the trains were 600 miles apart. How fast was each train traveling?

Week 3: Properties of Exponents

Properties of Exponents

Assume that a, b, m, n are real numbers.

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad \frac{1}{a^{-n}} = a^n$$

$$a^m a^n = a^{m+n} \qquad (a^m)^n = a^{m \cdot n} \qquad (ab)^n = a^n b^n$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example:

Simplify and rewrite each expression using only positive exponents.

a. $(5a^3)(-3a^{-4})$

$= 5(-3)a^3a^{-4}$

$= -15a^{[3+(-4)]}$

$= -15a^{-1}$

$= -\frac{15}{a}$

b. $\frac{4ab^6c^3}{a^5bc^3}$

$= 4a^{(1-5)}b^{(6-1)}c^{(3-3)}$

$= 4a^{-4}b^5c^0$

$= \frac{4b^5}{a^4}$

Simplify each expression. Use only positive exponents.

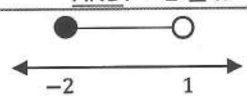
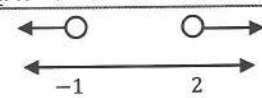
1. $(2a^3)(5a^4)$
2. $(-3x^2)(-4x^{-2})$
3. $(3x^2y^3)^2$
4. $(3x^{-4}y^3)^2$
5. $\frac{4a^8}{2a^4}$
6. $\frac{12x^5y^3}{4x^{-1}}$
7. $\frac{(6x^3)^0}{3xy^2}$
8. $\left(\frac{2x^4}{3}\right)^3$

8. $\left(\frac{2x^4}{3}\right)^3$

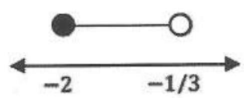
9. $(-4m^2n^3)(2mn)$
10. $(2x^3y^7)^{-2}$
11. $(h^7k^3)^0$
12. $\frac{x^2y}{4} \cdot \frac{16x}{y}$
13. $(s^4t)^2(st)$

Week 4: Solving Linear Inequalities

*Remember, when you multiply or divide each side of an inequality by a negative, you must reverse the inequality symbol.
*Closed dot represents $\geq$ and $\leq$. This means the value is included in the solution.
*Open dot represents $>$ and $<$. This means our value is not included in the solution.
Compound Inequalities: Two simple inequalities joined by the words "and" or "or".

AND: $-2 \leq x < 1$	OR: $x < -1$ or $x \geq 2$
	

Example:

$3 < -6x + 1 \leq 13$	Original Problem
$3 < -6x + 1 \leq 13$ $-1 \quad\quad -1 \quad -1$	Subtract 1 on each side
$\frac{2}{-6} < \frac{-6x}{-6} \leq \frac{12}{-6}$	Divide each side by -6 **Remember to reverse each inequality sign
$-\frac{1}{3} > x \geq -2$	Final Answer
	Graph

Write the inequality that represents the sentence.

1. Five less than a number is at least -28.
2. The product of a number and four is at most -10.
3. Six more than a quotient of a number and three is greater than 14.

Solve each inequality. Graph the solution.

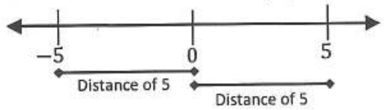
4. $5a - 10 > 5$
5. $25 - 2y \geq 33$
6. $-2(n + 2) + 6 \leq 16$
7. $2(7a + 1) > 2a - 10$

Solve each compound inequality. Graph the solution.

8. $-8 < 4x < 12$
9. $-2 \leq 3x - 8 \leq 10$
10. $2x + 3 < 12$ or $4x - 7 > 21$
11. $2x > 3 - x$ or $2x < x - 3$

Week 5: Solving Absolute Value Equations and Inequalities

*The absolute value of a number x is written $|x|$, is the distance the value is from zero. The absolute value of a number is always positive. $|5| = |-5| = 5$



Example:
Solve the absolute value equation.

$$|2x - 5| = 9$$

Break original problem into two since the value of $2x - 5$ could have been a positive 9 or -9

Solve each smaller equation as normal

$2x - 5 = 9$ → $x = 7$

$2x - 5 = -9$ → $x = -2$

Always check answers in the original equation to make sure they make true statements. If an answer comes from correct algebra, but does not work in the original equation, it is called an ***extraneous solution*.

*To solve an absolute value inequality, it is important to remember that a $<$ or $\leq$ represents an AND statement and a $>$ or $\geq$ represents an OR statement.

Example

$$|2x + 7| < 11$$
$$-11 < 2x + 7 < 11$$
$$-18 < 2x < 4$$
$$-9 < x < 2$$

Example

$$|3x - 2| \geq 8$$

$3x - 2 \leq -8$ Or $3x - 2 \geq 8$

$x \leq -2$ Or $x \geq \frac{10}{3}$

Solve each equation. Check for extraneous solutions.

1. $|-3x| = 18$
2. $|t + 5| = 8$
3. $3|z + 7| = 12$
4. $|4 - 2y| + 5 = 9$

Solve each inequality. Graph the solution.

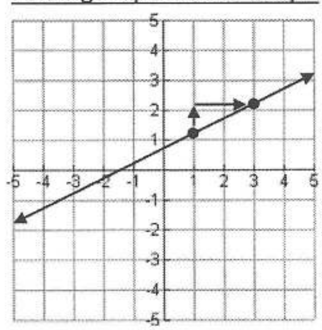
5. $5|y + 3| < 15$
6. $|4b| - 3 > 9$
7. $\frac{1}{2}|2x - 1| - 3 \geq 1$
8. $2|4x + 1| - 5 \leq 1$

Week 6 Lines

Finding Slope m

$$m = \frac{\Delta y}{\Delta x} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding Slope from a Graph



$\Delta y = 1$ Positive because it went up

Δx=2 Positive because it went right

Therefore, $m = \frac{1}{2}$.

Finding Slope from Two Points

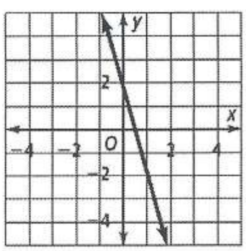
Example: Find the slope from the two points $(-2,7)$ and $(3,-1)$

$$\frac{-1-7}{3-(-2)} = \frac{-8}{5}$$

$$m = -\frac{8}{5}$$

Find the slope from the following lines or points.

1.



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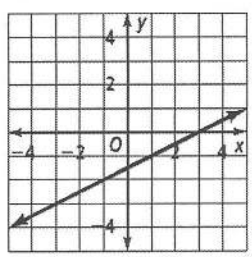

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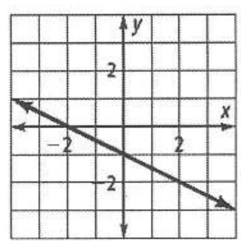

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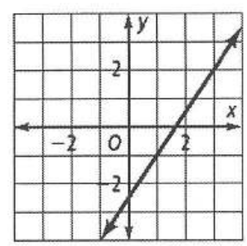


5. $(8,10)$, $(-7,14)$

   $m =$__________

6. $(-19,6)$, $(15,16)$

   $m =$__________

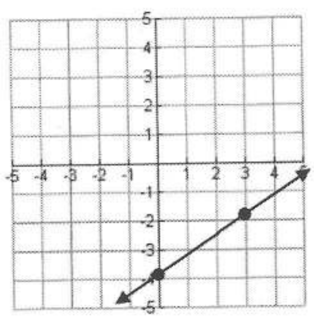
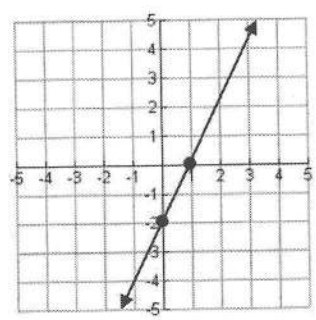
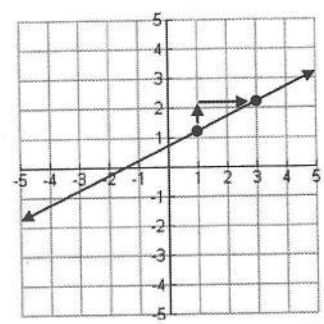
7. $(-18,-20)$, $(-18,5)$

   $m =$__________

8. $(4,7)$, $(8,7)$

   $m =$__________

Week 7: Graphing Linear Equations

Slope-Intercept Form	Standard Form	Point-Slope Form
$y = mx + b$ m is slope of the line b is the y-intercept $(0, b)$	$Ax + By = C$ x-intercept is $\frac{C}{A}$ (where $y = 0$) y-intercept is $\frac{C}{B}$ (where $x = 0$) Graph both intercepts and connect with a line	$y - y_1 = m(x - x_1)$ m is the slope of the line (x_1, y_1) is a point on the line Graph the point, then use the slope to graph more points
Example: Graph $y = \frac{2}{3}x - 4$ y-intercept is -4 or $(0, -4)$ slope is $\frac{2}{3}$ → move up 2, right 3	Example: Graph $-2x + y = -2$ x-intercept is $\frac{-2}{-2} = 1$ → $(1,0)$ y-intercept is $\frac{-2}{1} = -2$ → $(0,-2)$	Example: Graph $y - 1 = \frac{1}{2}(x - 1)$ Point: $(1,1)$ Slope: $\frac{1}{2}$ → move up 1, right 2
		

Identifying the x- and y-intercepts in Any Form

x-intercept is the point on the x-axis where the graph crosses. This is also the line where $y = 0$. Substitute $y = 0$ to find the x-value of this point.

y-intercept is the point on the y-axis where the graph crosses. This is also the line where $x = 0$. Substitute $x = 0$ to find the x-value of this point.

Example:

Find the x- and y-intercepts. $y - 3 = 3(x + 1)$

x-intercept: $y = 0$: $0 - 3 = 3(x + 1)$

Solve $-3 = 3x + 3$

$-6 = 3x$

$x = -2$

x-intercept: $(-2,0)$

y-intercept: $x = 0$: $y - 3 = 3(0 + 1)$

$y - 3 = 3(1)$

$y - 3 = 3$

$y = 6$

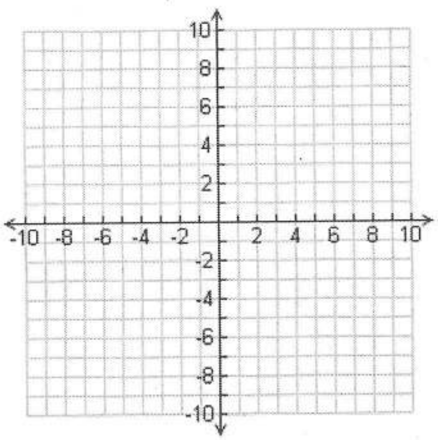
y-intercept: $(0,6)$

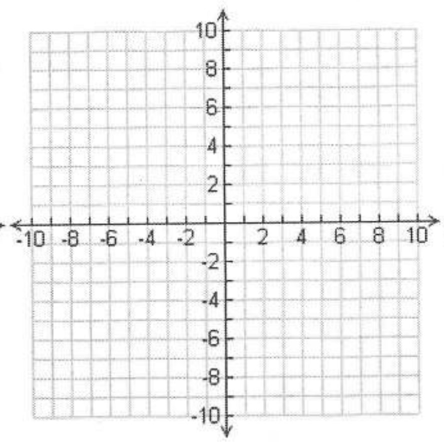
Graph each equation. Fill-in the appropriate information.

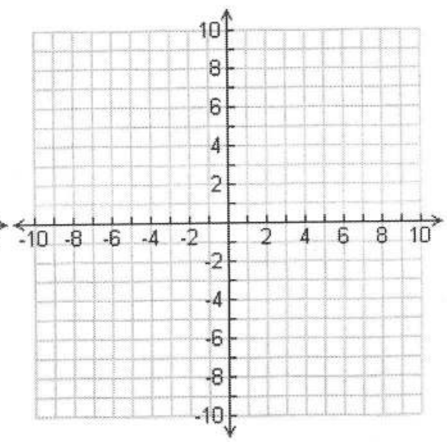
1. $x + y = 3$
x-intercept:__________
y-intercept:__________
Slope:_______________

2. $y = -2x - 3$
x-intercept:__________
y-intercept:__________
Slope:_______________

3. $y = 5x - 2$
x-intercept:__________
y-intercept:__________
Slope:_______________



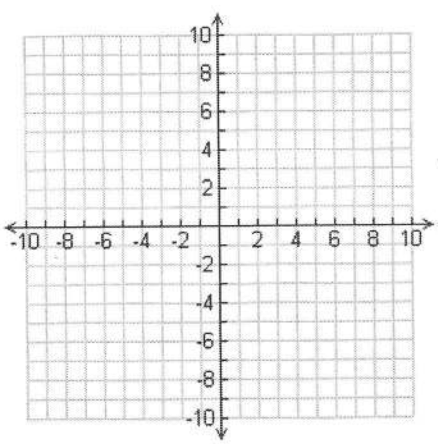


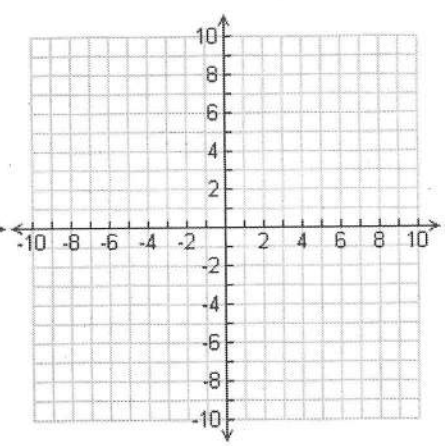


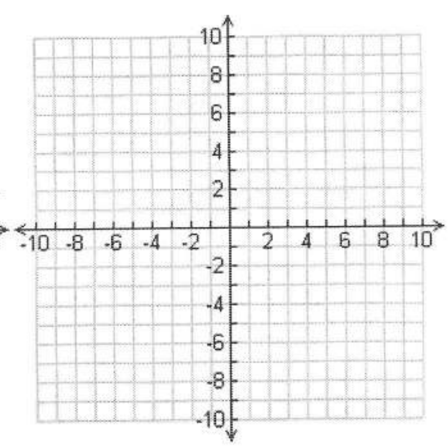
4. $y - 4 = \frac{1}{2}(x + 3)$
Point:_______________
Slope:_______________
x-intercept:__________
y-intercept:__________

5. $y - 5 = 2(x - 3)$
Point:_______________
Slope:_______________
x-intercept:__________
y-intercept:__________

6. $x + 4y = 4$
x-intercept:__________
y-intercept:__________
Slope:_______________







Week 8: Graphing Linear Inequalities

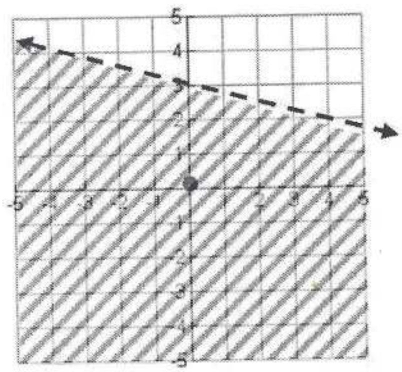
1. Graph the line the same way you would any other linear equation.
2. Remember $<$ or $>$ represents a dashed line and $\leq$ or $\geq$ represents a solid line.
3. Choose a test point on the graph to see if it satisfies the inequality. If it does, shade to cover the test point as it is a solution. If it is not, shade away from it.

Example: Graph $y < -\frac{1}{4}x + 3$

1. Graph $y = -\frac{1}{4}x + 3$
2. Graph has a dashed line.
3. Test Point: (0,0): $0 < -\frac{1}{4}(0) + 3$

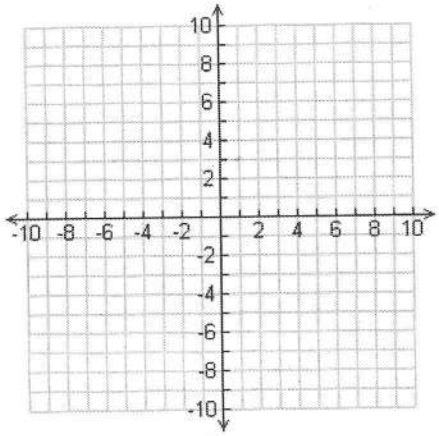
   $0 < 3$ → TRUE

   Shade to cover (0,0).

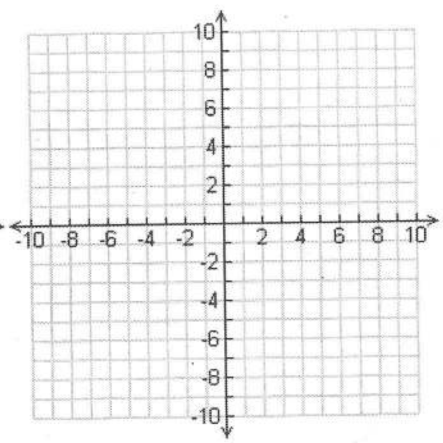


Graph the linear inequalities.

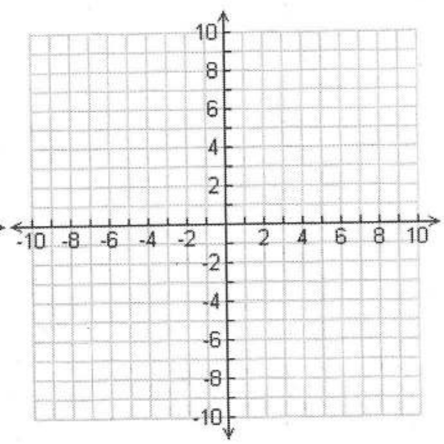
1. $3x + y \geq 6$



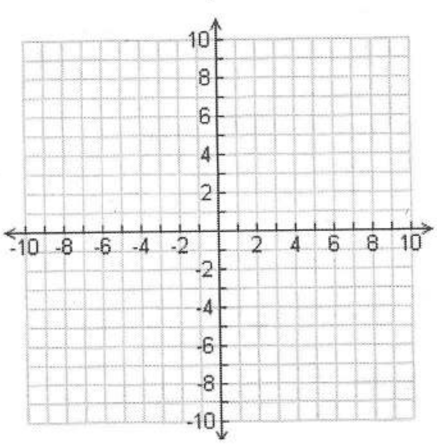
2. $x + y < -2$



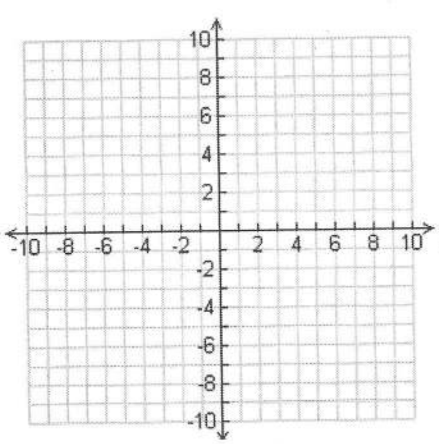
3. $x + 4y \leq 8$



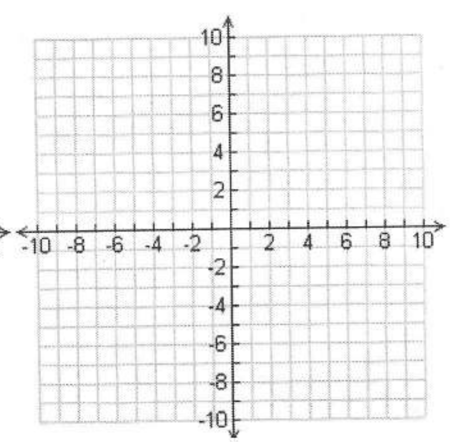
4. $y \leq \frac{3}{4}x + 1$



5. $y < -x + 4$



6. $y \geq -\frac{2}{5}x - 2$



Name ______________________ Class ____________ Date ____________

Week 9: Solving Systems of Equations

3-2 Reteaching

Solving Systems Algebraically

Follow these steps when solving by substitution.

Step 1 Solve one equation for one of the variables.

Step 2 Substitute the expression for this first variable into the other equation. Solve for the second variable.

Step 3 Substitute the second variable's value into either equation. Solve for the first variable.

Step 4 Check the solution in the other original equation.

Problem

What is the solution of the system of equations? $\begin{cases} 4x + 3y = 10 \\ x + 2y = 10 \end{cases}$

Step 1 $x = -2y + 10$ — Solve one equation for x.

Step 2

$4(-2y + 10) + 3y = 10$ — Substitute the expression for x into the other equation.

$-8y + 40 + 3y = 10$ — Distribute.

$-5y = -30$ — Combine like terms.

$y = 6$ — Solve for y.

Step 3

$x + 2(6) = 10$ — Substitute the y value into either equation.

$x + 12 = 10$ — Simplify.

$x = -2$ — Solve for x.

Step 4

$4(-2) + 3(6) \stackrel{?}{=} 10$ — Check the solution in the other equation.

$-8 + 18 \stackrel{?}{=} 10$ — Simplify.

$10 = 10$ ✓

The solution is $(-2, 6)$.

Exercises

Solve each system by substitution.

1. $\begin{cases} x - 3y = 2 \\ -x + 2y = 5 \end{cases}$

2. $\begin{cases} a + 3b = 4 \\ a = -2 \end{cases}$

3. $\begin{cases} -2m + n = 6 \\ -7m + 6n = 1 \end{cases}$

4. $\begin{cases} 7x - 3y = -1 \\ x + 2y = 12 \end{cases}$

Name ______________________ Class ____________ Date ____________

3-2 Reteaching (continued)

Solving Systems Algebraically

Follow these steps when solving by elimination.

Step 1 Arrange the equations with like terms in columns. Circle the like terms for which you want to obtain coefficients that are opposites.

Step 2 Multiply each term of one or both equations by an appropriate number.

Step 3 Add the equations.

Step 4 Solve for the remaining variable.

Step 5 Substitute the value obtained in step 4 into either of the original equations, and solve for the other variable.

Step 6 Check the solution in the other original equation.

Problem

What is the solution of the system of equations? $\begin{cases} 2x + 5y = 11 \\ 3x - 2y = -12 \end{cases}$

Step 1 $\begin{aligned} \textcircled{2x} + 5y &= 11 \\ \textcircled{3x} - 2y &= -12 \end{aligned}$ Circle the terms that you want to make opposite.

Step 2 $\begin{aligned} 6x + 15y &= 33 \\ -6x + 4y &= 24 \end{aligned}$ Multiply each term of the first equation by 3. Multiply each term of the second equation by -2.

Step 3 $19y = 57$ Add the equations.

Step 4 $y = 3$ Solve for the remaining variable.

Step 5 $\begin{aligned} 3x - 2(3) &= -12 \\ x &= -2 \end{aligned}$ Substitute 3 for y to solve for x.

Step 6 $\begin{aligned} 2(-2) + 5(3) &\stackrel{?}{=} 11 \\ -4 + 15 &\stackrel{?}{=} 11 \\ 11 &= 11 \checkmark \end{aligned}$ Check using the other equation.

The solution is $(-2, 3)$. You can also check the solution by using a graphing calculator.

Exercises

Solve each system by elimination.

5. $\begin{cases} 3x + 2y = -17 \\ x - 3y = 9 \end{cases}$

6. $\begin{cases} 5f + 4m = 6 \\ -2f - 3m = -1 \end{cases}$

7. $\begin{cases} 3x - 2y = 5 \\ -6x + 4y = 7 \end{cases}$

8. $\begin{cases} -2x - 4y = 2 \\ 10x + 20y = -10 \end{cases}$

9. **Reasoning** Why does a system with no solution represent parallel lines?