Discrete Math Summer Review Packet

Algebra – Things to Remember!

Scientific Notation:
3.2×10^{13}
The first number must be $1 \leq n < 10$

Factorial:
$5! = 5\cdot4\cdot3\cdot2\cdot1$
$1! = 1$
FYI: $0! = 1$

Absolute Value:
$|-5| = 5$
$|5| = 5$
Represents distance

Undefined:
$\frac{6}{7-x}$ is undefined when $x = 7$ since the denominator = 0.

Multiply: (distribute or FOIL)
$(x+3)(x+2) = x\cdot x + x\cdot 2 + 3\cdot x + 3\cdot 2$
$= x^2 + 5x + 6$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Add Fractions:
Get the common denominator:
$\frac{5x}{6} + \frac{3x}{2} = \frac{5x}{6} + \frac{9x}{6} = \frac{14x}{6} = \frac{7x}{3}$

Inequalities:
$5 - 3x \leq 13 + x$ Remember to
$-3x \leq 8 + x$ change direction
$-4x \leq 8$ of inequality when
$x \geq -2$ mult/div by a negative.

x = abscissa, y = ordinate

Slope:
$m = \frac{\textit{vertical change}}{\textit{horizontal change}} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

Exponents:
$(-3)^2 \neq -3^2$
$2^0 = 1$
$4^{-3} = \frac{1}{4^3}$
$x^m \cdot x^n = x^{m+n}$
$(x^n)^m = x^{n\cdot m}$
$\frac{x^m}{x^n} = x^{m-n}$
$(xy)^n = x^n \cdot y^n$

Polygons and sides:
triangle – 3
quadrilateral – 4
pentagon – 5
hexagon – 6
septagon – 7
octagon – 8
nonagon – 9
decagon – 10
dodecagon - 12

Direct Variation:
$y = kx$ where k = constant of variation
$k = y/x$

Factor:
Look for a GCF (greatest common factor)
Factor binomial or trinomial.
$a^2 - b^2 = (a+b)(a-b)$

Systems:

$y - 2x = 1$ $y + 2x = 9$	*Linear:* substitute; add to eliminate one variable or graph.
$y = x^2 - x - 6$ $y = 2x - 2$	*Linear Quadratic:* substitute or graph

For inequality systems, graph.

Equations of Lines: m = slope
$y = mx + b$ slope-intercept
$y - y_1 = m(x - x_1)$ point-slope

Properties of Real Numbers:

Commutative Property:	a + b = b + a	ab = ba
Associative Property:	a+(b+c) = (a+b)+c	a(bc) = (ab)c
Distributive Property:	a(b+c) = ab + ac	
Identity:	a + 0 = a	a • 1 = a
Inverse:	a + (-a) = 0	a • (1/a) = 1
Zero Property:		a • 0 = 0

Degree:
Degree of monomial = sum of exponents
$4x^3$ is of degree 3
x^2y^3 is of degree 5

Solving Equations:
1. Deal with any parentheses in the problem.
2. Combine similar terms on same side of = sign.
3. Get the needed variables on the same side of = sign.
4. Isolate the needed variable by add or subtract.
5. Find the needed variable by divide or multiply.

Quadratic Equation:
$x^2 - 5x + 6 = 0$ Set = 0.
$(x-3)(x-2) = 0$ Factor.
$x = 3;\ x = 2$ Find roots

Interval Notation:
$(1,5) \leftrightarrow 1 < x < 5$
$[1,5] \leftrightarrow 1 \leq x \leq 5$

Function: Passes the vertical line test.
A set of ordered pairs in which each x element has only one y element associated with it.
$f(x) = 3x + 4$
$f(3) = 3\cdot3 + 4 = 13$

Parallel and Perpendicular:
Parallel: slopes are equal.
Perpendicular: slopes are negative reciprocals (flip over and negate)

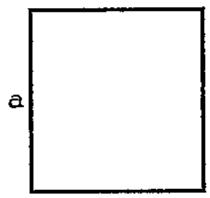
Parabola:
$y = ax^2 + bx + c$
Axis of symmetry:
$x = \frac{-b}{2a}$
Roots: where the graph crosses the x-axis.

GEOMETRY FORMULAS

SHAPES — perimeter (P) and area (A)

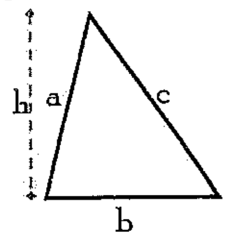
SQUARE
$P = 4a$

$A = a^2$



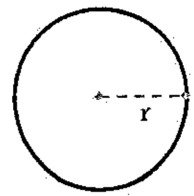
TRIANGLE
$P = a+b+c$

$A = \frac{1}{2}bh$



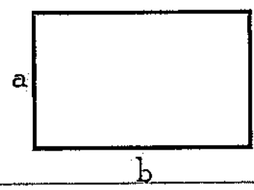
CIRCLE
$C = 2\pi r$

$A = \pi r^2$



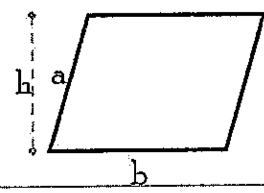
RECTANGLE
$P = 2a+2b$

$A = ab$



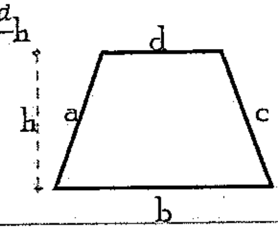
PARALLELOGRAM
$P = 2a+2b$

$A = bh$



TRAPEZOID
$P = a+b+c+d$

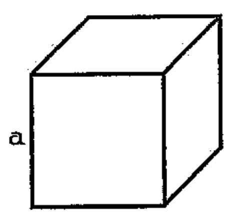
$A = \frac{b+d}{2}h$



SOLIDS — surface area (SA) and volume (V)

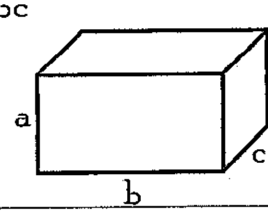
CUBE
$SA = 6a^2$

$V = a^3$



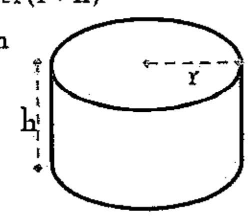
RECTANGULAR PRISM
$SA = 2ab+2ac+2bc$

$V = abc$



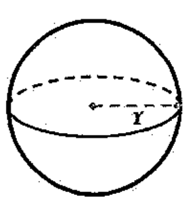
CYLINDER
$SA = 2\pi r(r+h)$

$V = \pi r^2 h$

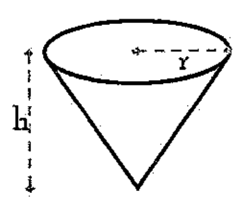


SPHERE
$SA = 4\pi r^2$

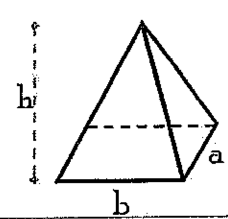
$V = \frac{4\pi r^3}{3}$



CONE
$V = \frac{1}{3}\pi r^2 h$



RECTANGULAR PYRAMID
$V = \frac{1}{3}abh$



Geometry
Algebra Review: 0-5 Linear Equations

If the same number is added to or subtracted from each side of an equation the resulting equation is true.

Example 1:

a. $x + 8 = -5$ b. $n - 15 = 3$ c. $p + 27 = 12$

If each side of an equation is multiplied or divided by the same number, the resulting equation is true.

Example 2:

a. $5g = 35$ b. $-\frac{c}{6} = 8$ c. $\frac{4x}{7} = -3$

To solve equations with more than one operation, often called **multi-step equations**, undo operations by working backward.

Example 3:

a. $9p + 8 = 35$ b. $8x + 2 = 14x - 7$

When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

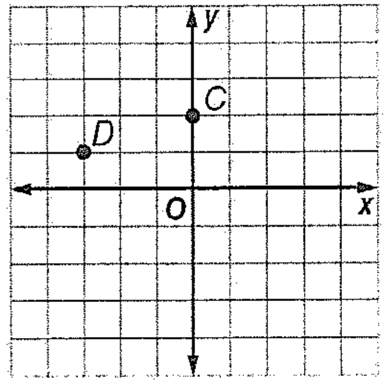
Example 4:

a. $4(x - 7) = 8x + 6$ b. $\frac{1}{3}(18 + 12x) = 6(2x - 7)$

Geometry
Algebra Review: 0-7 Ordered Pairs

Points in the coordinate plane are named by **ordered pairs** of the form (x, y). The first number, or **x-coordinate**, corresponds to a number on the x-axis. The second number, or **y-coordinate**, corresponds to a number on the y-axis.

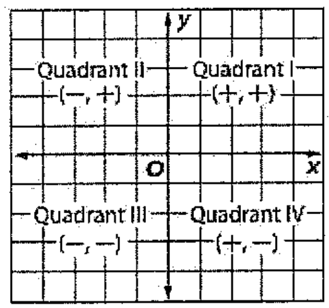
Example 1: Write the ordered pair for each point.



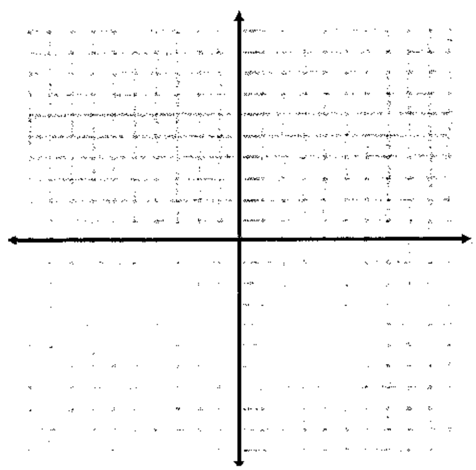
a. Point C

b. Point D

The x-axis and y-axis separate the coordinate plane into four regions, called **quadrants**. The point at which the axis intersect is called the **origin**. The axes and points on the axes are not located in any of the quadrants.



Example 2: Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

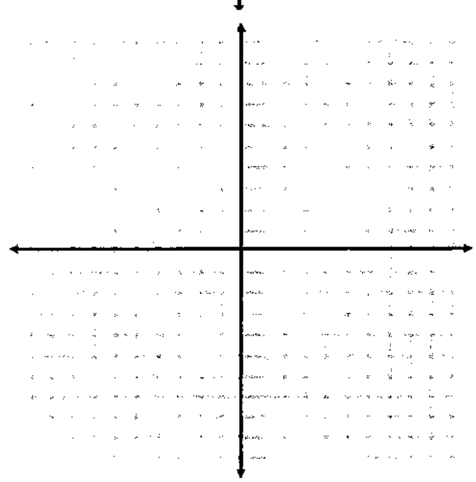


a. P(4, 2)

b. M(−2, 4)

c. N(−1, 0)

Example 3:
Graph a polygon with vertices P(−1, 1), Q(3, 1), R(1, 4), and S(−3, 4).

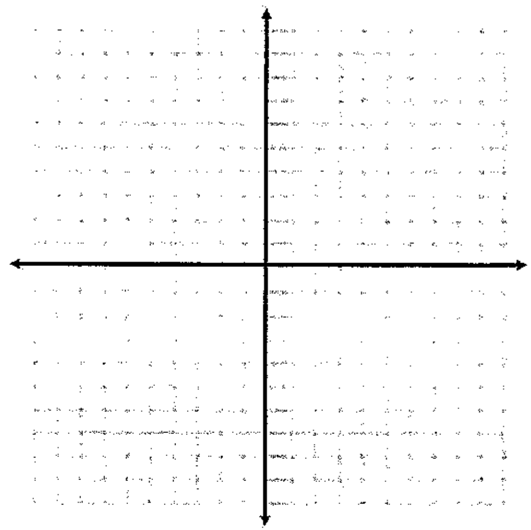


Remember lines have infinitely many points on them. So when you are asked to find points on a line, there are many answers.

*Make a table. Choose values for x. Evaluate each value of x to determine the y. Plot the ordered pairs.

Example 4:

Graph four points that satisfy the equation $y = -x - 2$

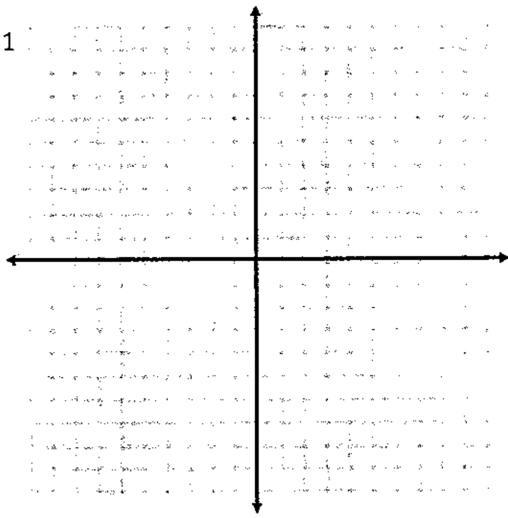


Geometry
Algebra Review: 0-8 Systems of Linear Equations

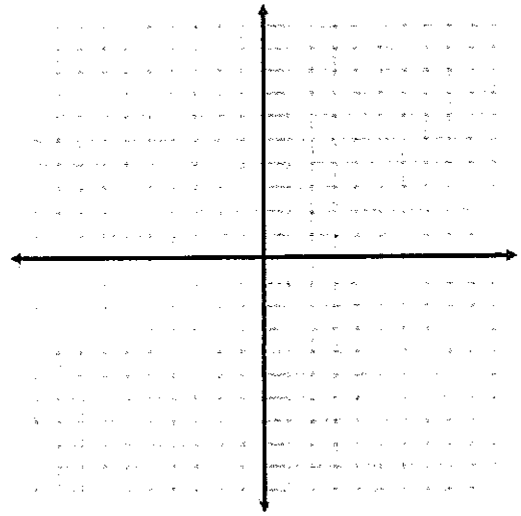
Two or more equations that have common variables are called **system of equations.** The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.

Example 1: Solve each system of equations by graphing. Then determine whether each system has *no solution*, *one solution*, or *infinitely many solutions.*

a. $y = -3x + 1$
$y = x - 3$



b. $y = 2x + 3$
$-4x + 2y = 6$



It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution.**

Example 2: Use substitution to solve the system of equations.

a. $y = 3x$
$-2y + 9x = 5$

b. $3x + 2y = 10$
$2x + 3y = 10$

Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination.**

Example 3: Use elimination to solve the system of equations.

a. $-3x + 4y = 12$
$3x - 6y = 18$

b. $3x + 7y = 15$
$5x + 2y = -4$

Geometry
Skills Review Worksheet

Name: ______________________________

For numbers 1 – 3, solve each equation.

1. $8x - 2 = -9 + 7x$

2. $12 = -4(-6x - 3)$

3. $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$

For numbers 4 – 6, simplify each expression by multiplying.

4. $2x(-2x - 3)$

5. $(8p - 2)(6p + 2)$

6. $(n^2 + 6n - 4)(2n - 4)$

For numbers 7 – 9, factor each expression.

7. $b^2 + 8b + 7$

8. $b^2 + 16b + 64$

9. $2n^2 + 5n + 2$

For numbers 10 – 14, solve each equation.

10. $9n^2 + 10 = 91$

11. $(k + 1)(k - 5) = 0$

12. $n^2 + 7n + 15 = 5$

13. $n^2 - 10n + 22 = -2$

14. $2m^2 - 7m - 13 = -10$

For numbers 15 – 18, simplify each radical.

15. $\sqrt{72}$

16. $\sqrt{80}$

17. $\sqrt{32}$

18. $\sqrt{90}$

Geometry
Algebra Skills Practice

Name: ______________________

I. Solving Linear Equations

1. $2x + 5 = 11$

2. $3x + 5 = -16$

3. $2(x - 3) = 84$

4. $5x - 32 = 80$

5. $3(2x + 5) - 3x = 6$

6. $3x - 4(x - 4) + 4 = 13$

II. Solving Systems of Equations by <u>Elimination.</u>

7. $\begin{cases} 2x + 7y = 3 \\ -4x - 2y = -18 \end{cases}$

8. $\begin{cases} x - y = 39 \\ x + y = 1785 \end{cases}$

9. $\begin{cases} 6x + 4y = 7 \\ 15x - 12y = 1 \end{cases}$

10. $\begin{cases} 11x - 3y = -39 \\ 6x + 12y = -19 \end{cases}$

III. Solving Systems of Equations by <u>Substitution</u>

11. $\begin{cases} x - 6y = -2 \\ -5x + 30y = 10 \end{cases}$

12. $\begin{cases} 9x - 2y = -6 \\ 5x + 4y = 12 \end{cases}$

13. $\begin{cases} 2x + 3y = 8 \\ 9x - 3y = 14 \end{cases}$

14. $\begin{cases} 10x - 5y = 3 \\ 6x + 30y = 81 \end{cases}$