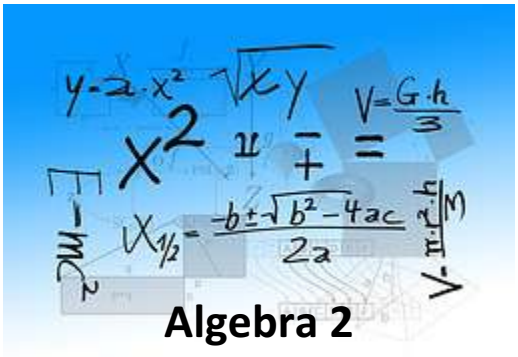
Regina Summer Math Review

For students who will be taking

Algebra 2

Completed review packet due the first day of classes



Algebra 2
Summer Review Packet

Welcome to Algebra 2!

In the following pages, you will find review materials that will prepare you for Algebra 2. Please take the exercises seriously as this will allow us to hit the ground running in the fall. If the examples preceding the practice problems are not enough of a reminder of a concept, please remember that Kahn Academy, YouTube, and math.com are very useful resources!

The review materials are separated into weeks. These weeks are only a suggestion. You will have the most benefit from this material if you work on it throughout the summer and do a final review of your work a week or two before school starts. **This packet must be complete the first day of class.**

Table of Contents:

Materials Needed for Algebra

- *Graphing calculator (Texas Instruments models TI-83, TI-84 families)
- 3-Ring Binder (Required)
- Loose-Leaf Paper (Required)

Graphing calculators are needed in this course and subsequent courses, even courses in college. Invest in one now, keep good care of it, and use it for many years to come.

Algebra 2 Summer Review

Week 1: Real numbers and Operations

Types of Real Numbers:

Natural (Counting): 1, 2, 3, 4, …
Whole: 0, 1, 2, 3, …
Integers: … -2, -1, 0, 1, 2, …
Rational: Can be written as a fraction or a repeating or terminating decimal
Irrational: Not rational

Properties of Addition and Multiplication (Addition shown first, Multiplication second)

1.	Commutative Property:	$a + b = b + a$	$a \cdot b = b \cdot a$
2.	Associative Property:	$(a + b) + c = a + (b + c)$	$a(b \cdot c) = (a \cdot b)c$
3.	Identity Property:	$a + 0 = a$	$a \cdot 1 = a$
4.	Inverse Property:	$a + (-a) = 0$	$a \cdot \frac{1}{a} = 1$
5.	Distributive Property:	$a(b + c) = ab + bc$	

Classify each variable according to the set of numbers that best describes its values.

1. the area of the circle *A* found by using the formula πr^2 1._____
2. the number *n* of equal slices in a pizza; the portion *p* of the pizza in one slice 2._____
3. the air temperature *t* in Saint Paul, MN, measured to the nearest degree Fahrenheit 3._____
4. the last four digits *s* of a Social Security number 4._____

Plot the following values on the number line provided. Please label each by its exercise number.

5. $5\frac{1}{2}$ 6. -4 7. 2.25 8. $-6\frac{1}{3}$ 9. $\sqrt{8}$

Compare the two numbers. Use >, <, or =. A calculator should not be used.

10. $-\sqrt{2}$ -2 11. $\sqrt{29}$ 5 12. 4 $\sqrt{17}$ 13. $\sqrt{50}$ 6.8

Name the property of real numbers illustrated by each equation.

14. $2(3 + \sqrt{5}) = 2 \cdot 3 + 2 \cdot \sqrt{5}$ 14._____
15. $16 + (-13) = -13 + 16$ 15._____
16. $-7 \cdot \left(-\frac{1}{7}\right) = 1$ 16._____
17. $5(0.2 \cdot 7) = (5 \cdot 0.2) \cdot 7$ 17._____

Week 2: Algebraic Expressions and Models

Order of Operations	Vocabulary
Parentheses Exponents Multiplication/Division Addition/Subtraction	Terms: Parts added together to make an expression. Coefficients: The number located in front of the variable. Constant: Numbers in an expression without a variable.

Write an algebraic expression that models each word phrase.

1. seven less than the number t 1._____
2. the sum of 11 and the product of 2 and a number r 2._____

Write an algebraic expression that models each situation.

3. Arin has $520 and is earning $75 each week babysitting. 1._____
4. You have 50 boxes of raisins and are eating 12 boxes each month. 2._____

Evaluate each expression for the given values of the variables.

5. $-4v + 3(w + 2v) - 5w \quad v = -2 \quad w = 4$ 5._____
6. $c(3 - a) - c^2 \quad a = 4 \quad c = -1$ 6._____
7. $2(3g - 5f) + 3(g + 4f) \quad g = 3 \quad f = -5$ 7._____

Simplify by combining like terms.

8. $5x - 3x^2 + 16x^2$
9. $\frac{3(a-b)}{9} + \frac{4}{9}b$
10. $t + \frac{t^2}{2} + t^2 + t$
11. $4a - 5(a + 1)$

8._____
9._____
10._____
11._____

Identify the following components from the expression $5x^7 - 8x + 47$

12. The number of terms 12._____
13. Leading coefficient 13._____
14. Constant Term 14._____

Week 3: Solving Linear Equations

Remember to solve equations, you can add, subtract, multiply or divide by any number or variable as long as you do the same operation to the other (entire) side.

Example:

$\frac{2}{5}(x-3) = x-2$	Original Problem
$\frac{2}{5}x - \frac{6}{5} = x-2$	Distribute the $\frac{2}{5}$ to each term on the left side of the =
$5\left(\frac{2}{5}x - \frac{6}{5}\right) = 5(x-2)$	Multiply both sides of the equation by 5 to get rid of the fractions on the left
$2x-6 = 5x-10$ $-5x \quad -5x$	Subtract $5x$ on each side
$-3x-6 = -10$ $+6 \quad +6$	Add 6 on each side
$\frac{-3x}{-3} = \frac{-4}{-3}$	Divide both sides by -3
$x = \frac{4}{3}$	Solve for x.

Check your solution by plugging the value into the original equation.

Solve each equation.

1. $9(z-3) = 12z$ 1._____
2. $7y+5 = 6y+11$ 2._____
3. $5w+8-12w = 16-15w$ 3._____
4. $3(x+1) = 2(x+11)$ 4._____
5. $\frac{1}{3}(y-2) = y+4$ 5._____
6. $4-\frac{2}{3}x = -7$ 6._____

Write an equation and solve each problem.

7. Two brothers are saving money to buy tickets to a concert. Their combined savings is $55. One brother has $15 more than the other. How much has each saved?

8. What three consecutive numbers have a sum of 126?

9. Two trains left a station at the same time. One traveled north at a certain speed and the other traveled south at twice that speed. After 4 hours, the trains were 600 miles apart. How fast was each train traveling?

Week 4: Properties of Exponents

Properties of Exponents

Assume that a, b, m, n are real numbers.

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad \frac{1}{a^{-n}} = a^n$$

$$a^m a^n = a^{m+n} \qquad (a^m)^n = a^{m \cdot n} \qquad (ab)^n = a^n b^n$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example:

Simplify and rewrite each expression using only positive exponents.

a. $(5a^3)(-3a^{-4})$

$= 5(-3)a^3 a^{-4}$

$= -15a^{[3+(-4)]}$

$= -15a^{-1}$

$= -\frac{15}{a}$

b. $\frac{4ab^6c^3}{a^5bc^3}$

$= 4a^{(1-5)}b^{(6-1)}c^{(3-3)}$

$= 4a^{-4}b^5c^0$

$= \frac{4b^5}{a^4}$

Simplify each expression. Use only positive exponents.

1. $(2a^3)(5a^4)$
2. $(-3x^2)(-4x^{-2})$
3. $(3x^2y^3)^2$
4. $(3x^{-4}y^3)^2$
5. $\frac{4a^8}{2a^4}$
6. $\frac{12x^5y^3}{4x^{-1}}$
7. $\frac{(6x^3)^0}{3xy^2}$
8. $\left(\frac{2x^4}{3}\right)^3$
9. $(-4m^2n^3)(2mn)$
10. $(2x^3y^7)^{-2}$
11. $\frac{(3r^{-2}s^3t^0)^{-3}}{3rs}$
12. $(h^7k^3)^0$
13. $\frac{r^2s^4t^6}{r^3s^4t^{-6}}$
14. $\frac{x^2y}{4} \cdot \frac{16x}{y}$
15. $(s^4t)^2(st)$

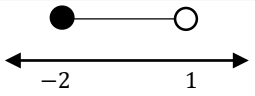
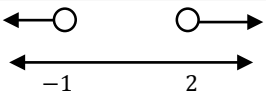
Week 5: Solving Linear Inequalities

*Remember, when you multiply or divide each side of an inequality by a negative, you must reverse the inequality symbol.

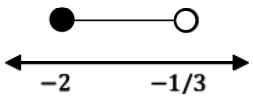
*Closed dot represents $\geq \quad and \quad \leq$. This means the value is included in the solution.

*Open dot represents $> \quad and \quad <$. This means our value is not included in the solution.

Compound Inequalities: Two simple inequalities joined by the words "and" or "or".

AND: $-2 \leq x < 1$	OR: $x < -1 \quad or \quad x \geq 2$
	

Example:

$3 < -6x + 1 \leq 13$	Original Problem
$3 < -6x + 1 \leq 13$ $-1 \quad -1 \quad -1$	Subtract 1 on each side
$\frac{2}{-6} < \frac{-6x}{-6} \leq \frac{12}{-6}$	Divide each side by -6 **Remember to reverse each inequality sign
$-\frac{1}{3} > x \geq -2$	Final Answer
	Graph

Write the inequality that represents the sentence.

1. Five less than a number is at least -28.
2. The product of a number and four is at most -10.
3. Six more than a quotient of a number and three is greater than 14.

Solve each inequality. Graph the solution.

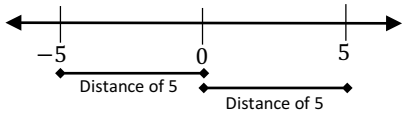
4. $5a - 10 > 5$
5. $25 - 2y \geq 33$
6. $-2(n + 2) + 6 \leq 16$
7. $2(7a + 1) > 2a - 10$

Solve each compound inequality. Graph the solution.

8. $-8 < 4x < 12$
9. $-2 \leq 3x - 8 \leq 10$
10. $2x + 3 < 12$ or $4x - 7 > 21$
11. $2x > 3 - x$ or $2x < x - 3$

Week 6: Solving Absolute Value Equations and Inequalities

*The absolute value of a number x is written $|x|$, is the distance the value is from zero. The absolute value of a number is always positive. $|5| = |-5| = 5$



Example:
Solve the absolute value equation.

$|2x - 5| = 9$

$2x - 5 = 9$ $\quad$ $2x - 5 = -9$

$x = 7$ $\quad$ $x = -2$

Break original problem into two since the value of $2x - 5$ could have been a positive 9 or -9

Solve each smaller equation as normal

Always check answers in the original equation to make sure they make true statements. If an answer comes from correct algebra, but does not work in the original equation, it is called an ***extraneous solution*.

*To solve an absolute value inequality, it is important to remember that a $<$ or $\leq$ represents an AND statement and a $>$ or $\geq$ represents an OR statement.

Example

$|2x + 7| < 11$

$-11 < 2x + 7 < 11$

$-18 < 2x < 4$

$-9 < x < 2$

Example

$|3x - 2| \geq 8$

$3x - 2 \leq -8$ Or $3x - 2 \geq 8$

$x \leq -2$ Or $x \geq \frac{10}{3}$

Solve each equation. Check for extraneous solutions.

1. $|-3x| = 18$
2. $|t + 5| = 8$
3. $3|z + 7| = 12$
4. $|4 - 2y| + 5 = 9$

Solve each inequality. Graph the solution.

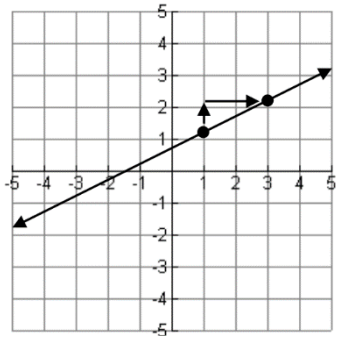
5. $5|y + 3| < 15$
6. $|4b| - 3 > 9$
7. $\frac{1}{2}|2x - 1| - 3 \geq 1$
8. $2|4x + 1| - 5 \leq 1$

Week 7: Lines

Finding Slope m

$$m = \frac{\Delta y}{\Delta x} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding Slope from a Graph



$\Delta y = 1$ Positive because it went up
Δx=2 Positive because it went right
Therefore, $m = \frac{1}{2}$.

Finding Slope from Two Points

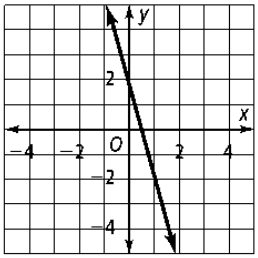
Example: Find the slope from the two points $(-2,7)$ and $(3,-1)$

$$\frac{-1-7}{3-(-2)} = \frac{-8}{5}$$

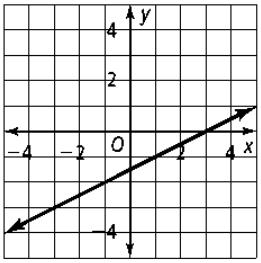
$$m = -\frac{8}{5}$$

Find the slope from the following lines or points.

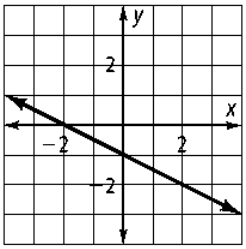
1.



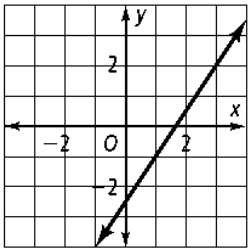
2.



3.



4.



5. $(8,10),\ (-7,14)$

$m =$_____

6. $(-19,6),\ (15,16)$

$m =$_____

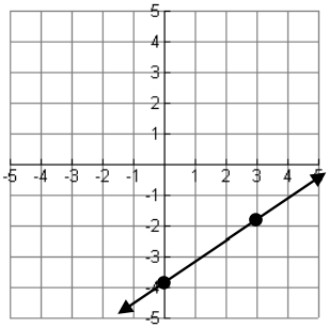
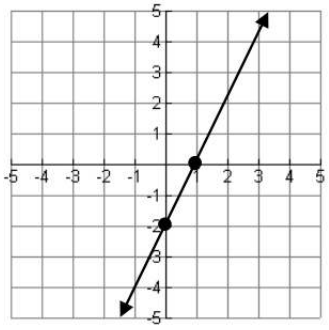
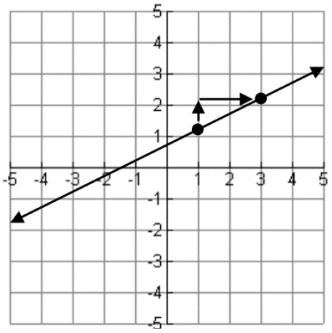
7. $(-18,-20),\ (-18,5)$

$m =$_____

8. $(4,7),\ (8,7)$

$m =$_____

Week 8: Graphing Linear Equations

Slope-Intercept Form	Standard Form	Point-Slope Form
$y = mx + b$	$Ax + By = C$	$y - y_1 = m(x - x_1)$
m is slope of the line	x-intercept is $\frac{C}{A}$ (where $y = 0$)	m is the slope of the line
b is the y-intercept $(0, b)$	y-intercept is $\frac{C}{B}$ (where $x = 0$)	(x_1, y_1) is a point on the line
	Graph both intercepts and connect with a line	Graph the point, then use the slope to graph more points
Example:	Example:	Example:
Graph $y = \frac{2}{3}x - 4$	Graph $-2x + y = -2$	Graph $y - 1 = \frac{1}{2}(x - 1)$
y-intercept is -4 or $(0, -4)$	x-intercept is $\frac{-2}{-2} = 1 \rightarrow (1,0)$	Point: $(1,1)$
slope is $\frac{2}{3} \rightarrow$ move up 2, right 3	y-intercept is $\frac{-2}{1} = -2 \rightarrow (0, -2)$	Slope: $\frac{1}{2} \rightarrow$ move up 1, right 2
		

Identifying the x- and y-intercepts in Any Form

x-intercept is the point on the x-axis where the graph crosses. This is also the line where $y = 0$. Substitute $y = 0$ to find the x-value of this point.

y-intercept is the point on the y-axis where the graph crosses. This is also the line where $x = 0$. Substitute $x = 0$ to find the x-value of this point.

Example:

Find the x- and y-intercepts. $y - 3 = 3(x + 1)$

x-intercept: $y = 0$: $0 - 3 = 3(x + 1)$

Solve $-3 = 3x + 3$

$-6 = 3x$

$x = -2$

x-intercept: $(-2,0)$

y-intercept: $x = 0$: $y - 3 = 3(0 + 1)$

$y - 3 = 3(1)$

$y - 3 = 3$

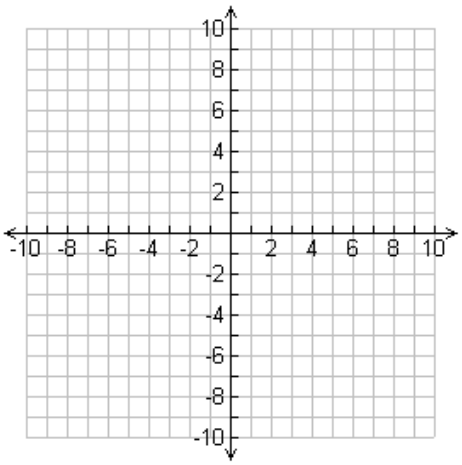
$y = 6$

y-intercept: $(0,6)$

Graph each equation. Fill-in the appropriate information.

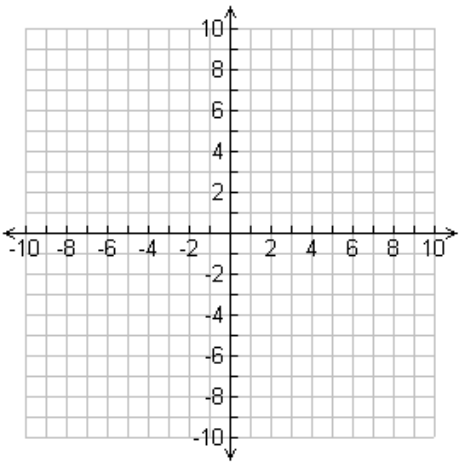
1. $x + y = 3$

x-intercept:_____
y-intercept:_____
Slope:_____



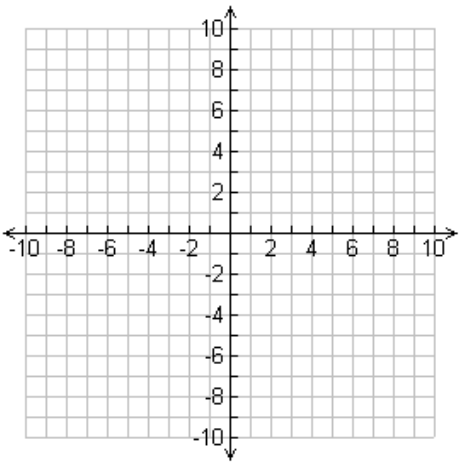
2. $y = -2x - 3$

x-intercept:_____
y-intercept:_____
Slope:_____



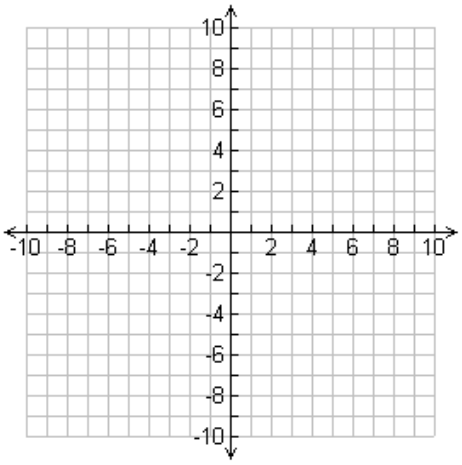
3. $y = 5x - 2$

x-intercept:_____
y-intercept:_____
Slope:_____



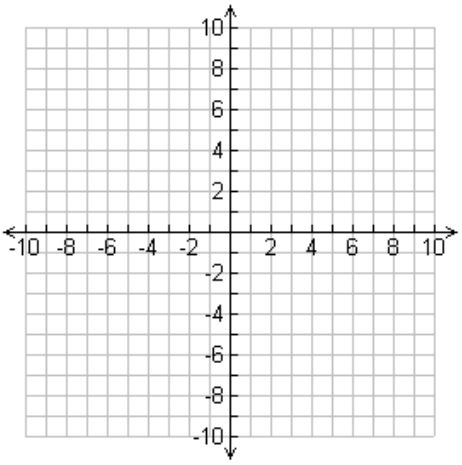
4. $y - 4 = \frac{1}{2}(x + 3)$

Point:_____
Slope:_____
x-intercept:_____
y-intercept:_____



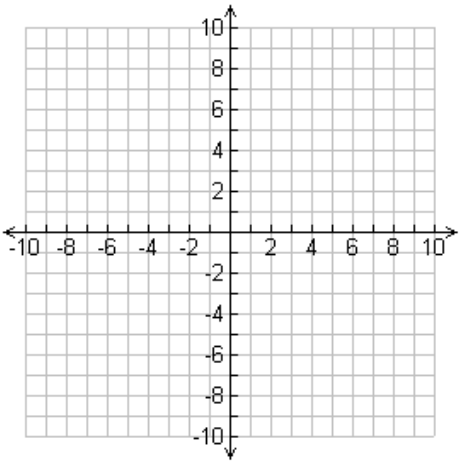
5. $y - 5 = 2(x - 3)$

Point:_____
Slope:_____
x-intercept:_____
y-intercept:_____



6. $x + 4y = 4$

x-intercept:_____
y-intercept:_____
Slope:_____



Week 9: Graphing Linear Inequalities

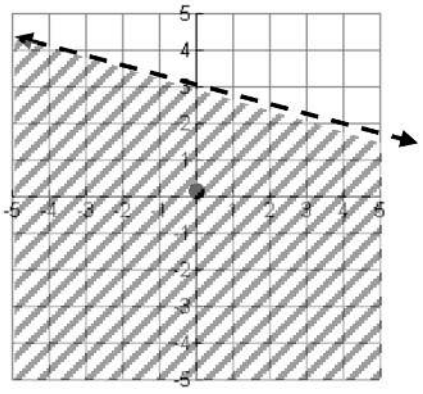
1. Graph the line the same way you would any other linear equation.
2. Remember $<$ or $>$ represents a dashed line and $\leq$ or $\geq$ represents a solid line.
3. Choose a test point on the graph to see if it satisfies the inequality. If it does, shade to cover the test point as it is a solution. If it is not, shade away from it.

Example: Graph $y < -\frac{1}{4}x + 3$

1. Graph $y = -\frac{1}{4}x + 3$
2. Graph has a dashed line.
3. Test Point: $(0,0)$: $0 < -\frac{1}{4}(0) + 3$

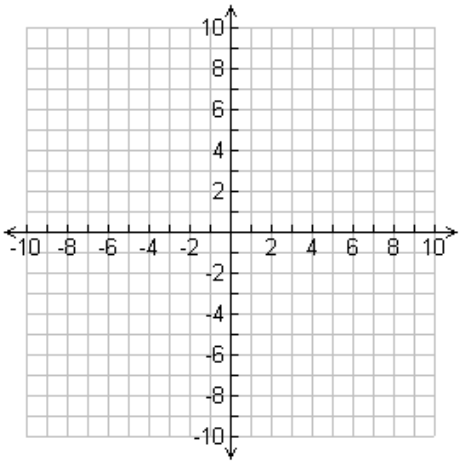
   $0 < 3$ → TRUE

   Shade to cover $(0,0)$.

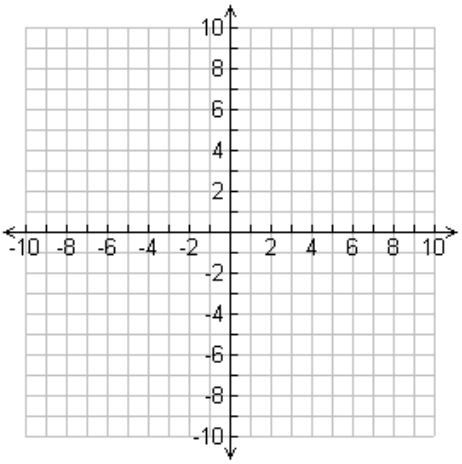


Graph the linear inequalities.

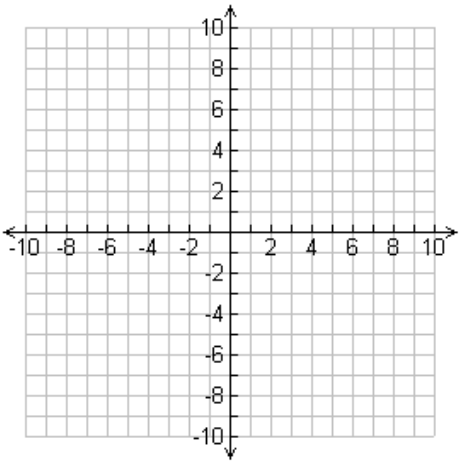
1. $3x + y \geq 6$



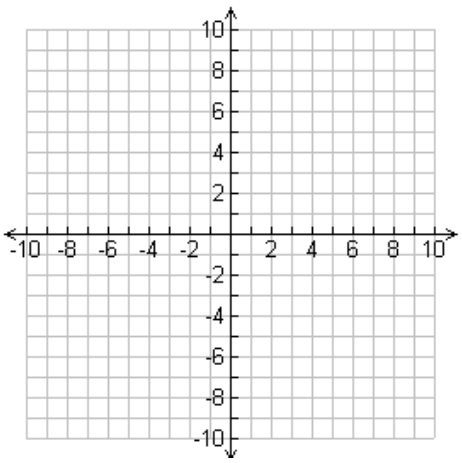
2. $x + y < -2$



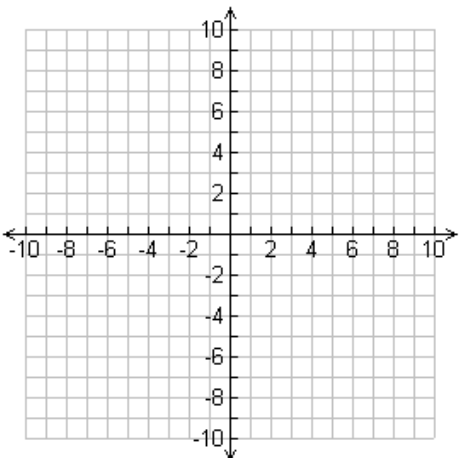
3. $x + 4y \leq 8$



4. $y \leq \frac{3}{4}x + 1$



5. $y < -x + 4$



6. $y \geq -\frac{2}{5}x - 2$