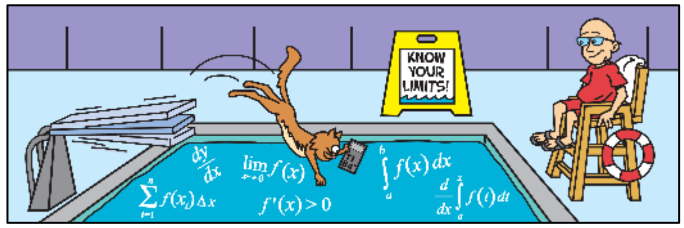
Regina Summer Math Review

For students who will be taking

AP Calculus BC

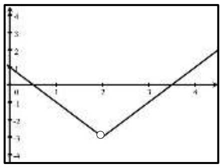
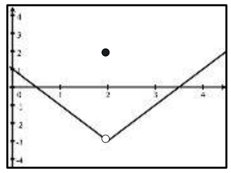
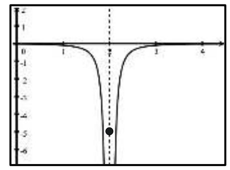
Completed review packet due the first day of classes

Diving In
The AP Calculus Exam



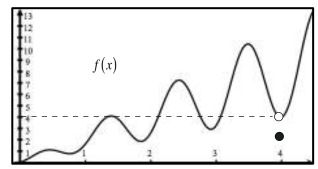
1) Limits

1. For which of the following functions f does $\lim_{x \to 2} f(x)$ exist?

I)  II)  III) 

B) I only R) II only S) III only T) I and II only Y) I, II and III

2. Using the graph of the function f to the right, which of the following is correct?



I. $\lim_{x \to 4} f(x) = 4$ II. $\lim_{x \to \infty} f(x) = \infty$ III. $\lim_{x \to \infty} f(x)$ does not exist

F) I only H) I and II only L) I and III only M) II and III only R) I, II and III

3. Find $\lim_{x \to \infty} \dfrac{2x}{\sqrt{x^2 + x + 100}} + \lim_{x \to -\infty} \dfrac{2x}{\sqrt{x^2 + x + 100}}$

A) 0 E) 2 I) 4 O) 400 U) does not exist

4. Find $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

C) 0 K) $\dfrac{1}{2}$ N) $\dfrac{1}{4}$ P) $\dfrac{1}{6}$ V) does not exist

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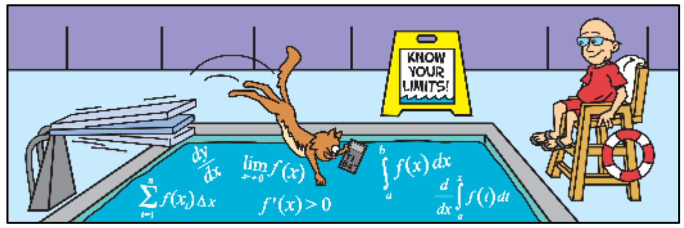

2) Rates of Change

1. Pretzels are given away for free at a local store. The tray is constantly replenished as people take the pretzels. The number of pretzels in the tray is given by the function $P(t) = 20 + t^2 - 8\cos \pi t$, where t is measured in minutes. When t = 1 minute, find the average rate of change of pretzels in the tray over the next 30 seconds.

D) $\dfrac{-3.88 \text{ pretzels}}{\text{min}}$ G) $\dfrac{-4.63 \text{ pretzels}}{\text{min}}$ J) $\dfrac{-6.75 \text{ pretzels}}{\text{min}}$ S) $\dfrac{-13.5 \text{ pretzels}}{\text{min}}$ W) $\dfrac{-18.5 \text{ pretzels}}{\text{min}}$

2. When the gates to a football game open, people who were waiting rush in. The rate R that people enter the stadium at various times is given in the chart below. Which of the following is an accurate statement?

time (sec)	0	30	60	90	120	150
R(people/sec)	90	240	300	360	350	240

I. At t = 1 minute, the instantaneous rate of change of R is approximately 2 people/sec.

II. The instantaneous rate of change of R at t = 15 seconds is approximately 5 people/sec^2.

III. At t = 90 seconds, more people are entering the stadium than at any other time over the first 2.5 minutes.

A) I only E) II only I) III only O) I and II only U) I, II and III

3. $f(x)$ is a function such that $f(0) = k$, $f(1) = 2k + 1$, and $f(5) = k^2 + 3k - 5$. If the average rate of change of f from x = 0 to x = 1 is the same as the average rate of change from x =1 to x = 5, find k.

D) k = –2 only K) k = 0 only M) k = 5 only P) k = –2, k = 5 Y) k = 0, k = 5

4. The velocity of an object is given by $v(t) = t^2 + 4t - 5$, t measured in seconds. Write an expression for the average rate of change of the object's velocity from 2 seconds to 2 + k seconds where $k \neq 0$.

C) k G) $2k$ H) $2 + k$ T) $8 + k$ W) $k - 2$

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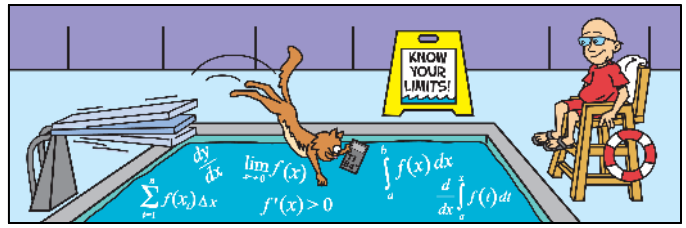

4) Basic Derivatives

1. If $f(x)=\dfrac{x^2-c^2}{x^2+c^2}$ where c is a constant, find $f'(x)$.

T) $\dfrac{4c^2x}{\left(x^2+c^2\right)^2}$ V) $\dfrac{-4c^2x}{\left(x^2+c^2\right)^2}$ W) $\dfrac{4cx(c-x)}{\left(x^2+c^2\right)^2}$ X) $\dfrac{x-c}{x+c}$ Y) 0

2. If $f(x)=x^2-x$ and $g(x)=\dfrac{1}{x}$, and $h(x)$ is defined with the expressions below, arrange the values of $h'(1)$ from smallest to largest.

I. $h(x)=f(x)-g(x)$ II. $h(x)=f(x)g(x)$ III. $h(x)=f(g(x))$

G) I, II, III H) I, III, II J) II, III, I K) III, I, II L) III, II, I

3. If $f(x)=x^3+2x^2-5x-1$, find $\lim\limits_{\Delta x\to 0}\dfrac{f'(-3+\Delta x)-f'(-3)}{\Delta x}$.

A) 10 E) 20 I) –14 O) –18 U) does not exist

4. If $f(x)=\dfrac{x}{x+1}$ and $\dfrac{1}{y}=f(x)+1$, find $\dfrac{dy}{dx}$.

A) $\dfrac{-1}{x^2}$ E) $\dfrac{x+1}{x^2}$ I) $\dfrac{4x+3}{2x+1}$ O) $\dfrac{4x+3}{(2x+1)^2}$ U) $\dfrac{-1}{(2x+1)^2}$

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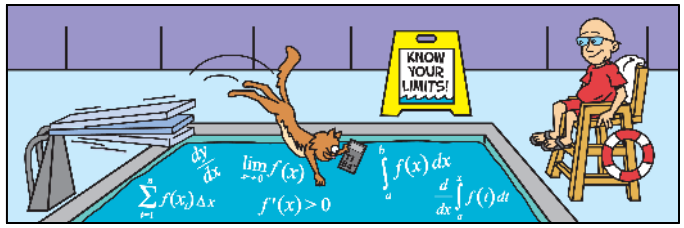
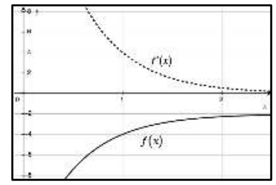


5) Equation of a Tangent Line

1. In the graph to the right, $f(x)$ is the solid curve and $f'(x)$ is the dashed curve. Find the equation of the tangent line to *f* at $x = 1$.



A) $y = 4$ E) $y = 4x - 8$ I) $y = 4x$ O) $y = \dfrac{-x-15}{4}$ U) $y = \dfrac{x+17}{4}$

2. If $f(x) = \dfrac{2x^2 - 6x}{\sqrt{x}}$, find the equation of the tangent line to *f* at $x = 9$.

M) $y = 12x - 72$ N) $y = 72x - 612$ P) $-8x + 108$ Q) $y = 8x - 36$ R) $8x - 9$

3. Given that $g(0) = 8$ and $g'(0) = -4$. Let *f* be a function such that $f'(x) = 2e^{x+1}\left[\dfrac{g(x)}{g'(x)}\right]$. Find the equation of the line tangent to *f* at its *y*-intercept of 4.

H) $y = -4x$ N) $y = 4x$ R) $y = 4(1 - ex)$ T) $y = 4 - ex$ Y) $y = 4 - 4e^x$

4. If $f(x) = \dfrac{x+1}{x-1}$, which of the following statements is true?

I. $\lim_{x\to 1} f(x) = -\infty$ II. $\lim_{x\to 1} f'(x) = -\infty$ III. $\lim_{x\to 1} f''(x) = -\infty$

S) I only T) II only V) III only W) I and II only X) II and III only

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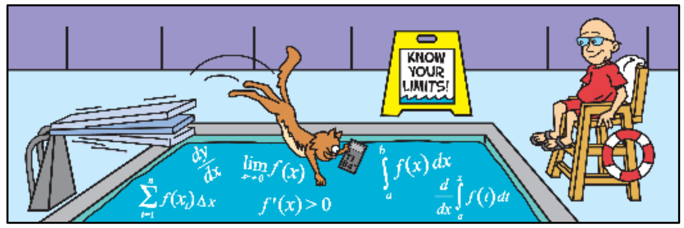


6) Derivatives of Trig Functions

1. Let $f(x) = \sin x \cos x + x$ for $0 \le x \le \frac{3\pi}{2}$. Find all values for which $f'(x) = 1$.

I. $x = \frac{\pi}{4}$ II. $x = \frac{3\pi}{4}$ III. $x = \frac{5\pi}{4}$

A) I only E) II only I) III only O) I, II, and III E) I and III only

2. If $g(x) = x^2 f'(x)$ where $f(x) = \frac{x - \cos x}{x}$, find the average rate of change of g on $[\pi, 2\pi]$.

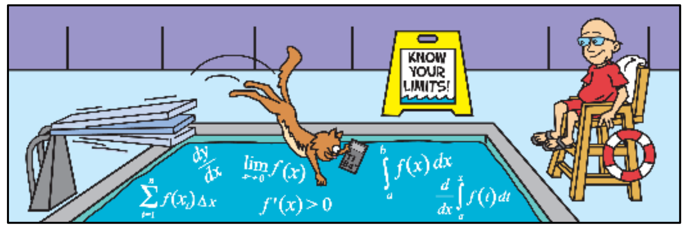
A) $\frac{-2}{\pi}$ E) $\frac{2}{\pi}$ I) -2 O) 2 U) 0

3. If $f(x) = \sec x$ and $k = \frac{7\pi}{4}$, arrange i) $f(k)$ ii) $f'(k)$ iii) $f''(k)$ from smallest to largest

F) i, ii, iii L) i, iii, ii N) ii, i, iii R) ii, iii, i S) iii, i, ii

4. Find $\lim\limits_{\Delta x \to 0} \dfrac{\csc\left(\frac{\pi}{6} + \Delta x\right) - \csc\frac{\pi}{6}}{\Delta x}$

A) $\frac{-2\sqrt{3}}{3}$ E) $\frac{2\sqrt{3}}{3}$ I) $\frac{\sqrt{3}}{2}$ O) $2\sqrt{3}$ U) $-2\sqrt{3}$



7) Chain Rule

1. If $y = x^2(1-2x)^3$, find y'.

A) $-x(2x-1)^2(x-2)$ E) $2x(1-2x)^2(5x-1)$ I) $-2x(2x-1)^2(5x-1)$

O) $3x^2(2x-1)^2$ U) $-6x^2(1-2x)^2$

2. The functions f and g are differentiable at $x = 10$ and $x = 20$ and $f(g(x)) = x^2$. If $f(10) = 5$, $f'(10) = 4$, $f'(20) = -5$, $g(10) = 20$, what is the value of $g'(10)$?

K) –2 L) –1 M) 1 N) –4 P) –5

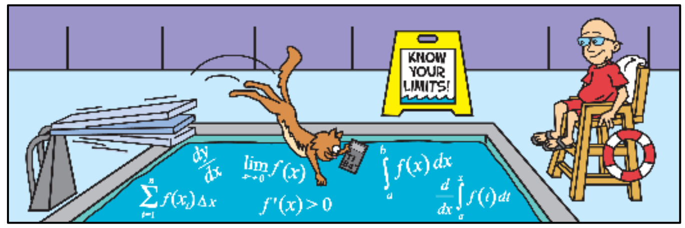
3. If $f(x) = \sin^2\left(2x + \frac{\pi}{4}\right)$, find $\lim\limits_{x \to \pi/4} \frac{f(x) - f(\pi/4)}{x - \pi/4}$.

R) $\frac{-\sqrt{2}}{2}$ S) $\frac{\sqrt{2}}{2}$ T) –2 V) 2 W) –1

4. In the table below, the values of $f(x)$, $g(x)$, $f'(x)$ and $g'(x)$ are given for two values of x. If $y = [f(2x) + g(x)]^2$, find $y'(3)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	4	–1	2	–5
6	–3	5	–2	–4

C) 0 D) –8 F) –12 G) 56 H) 72

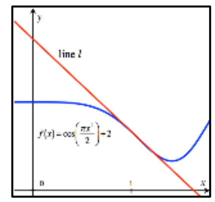


8) Local Linear Approximation

1. For the function *f*, $f(5) = -3$ and $f'(x) = 2x - 4$. What is the approximation for $f(5.2)$ found by using the tangent line to the graph of *f* at $x = 5$?

H) –1.8 J) –1.72 K) –1 L) 4.2 M) 6.4

2. The figure to the right (not to scale) shows the graph of $f(x) = \cos\left(\frac{\pi x^2}{2}\right) + 2$ and the line *l* tangent to *f* at $x = 1$. Find the *x*-intercept of line *l*.



A) 2 E) $\frac{2+\pi}{\pi}$ I) $\frac{2+\pi}{2}$ O) 3 U) $\frac{2+\pi}{2\pi}$

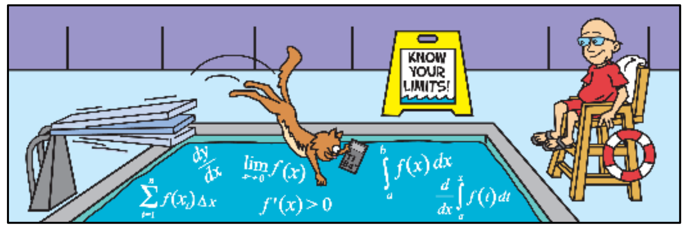
3. Given a function $f(x)$, what is the equation of the line normal to *f* at $x = a$?

A) $y = f(a) - f'(a)(x-a)$ E) $y = f(a) - \frac{1}{f'(a)}(x-a)$ I) $y = f(a) + \frac{1}{f'(a)}(x-a)$

O) $y = f'(a)(x-a) - f(a)$ U) $y = \frac{1}{f'(a)}(x-a) - f(a)$

4. For the differentiable function *f*, $f(2) = 5$ and $f'(2) = -3$. Function *f* is approximated using its tangent line at $x = 2$. For what value of *k* is the approximation to $f(k)$ equal to *k*?

G) $\frac{1}{4}$ H) $\frac{-1}{2}$ J) $\frac{11}{2}$ K) $\frac{11}{-2}$ L) $\frac{11}{4}$



9) Implicit Differentiation

1. For the curve $x^3 + y^2 = x + y$ passing through (–2, 3), use the tangent line at $x = -2$ to approximate the value of y at $x = -1.5$

A) $\dfrac{-157}{10}$ E) $\dfrac{-41}{10}$ I) $\dfrac{4}{5}$ O) $\dfrac{26}{5}$ U) $\dfrac{19}{10}$

2. For the curve $\sqrt{xy} - x + y = 1$, find the value of $\dfrac{dy}{dx}$ at the point (9, 4).

P) $\dfrac{-4}{9}$ R) $\dfrac{8}{21}$ S) $\dfrac{11}{12}$ T) $\dfrac{-8}{3}$ Y) $\dfrac{8}{3}$

3. For the graph of $\cos y = x - 4y$, what is the range of the slopes of its tangent lines?

B) $\left[0, \dfrac{1}{2}\right]$ C) $\left[\dfrac{1}{5}, \dfrac{1}{3}\right]$ D) $[3, \infty)$ F) $(-\infty, -3]$ G) $(-\infty, \infty)$

4. If $x^2 + y^2 = a$ where a is a non-zero constant, which of the following conditions are necessary for $\dfrac{d^2y}{dx^2} > 0$?

I. $y < 0$ II. $y > 0$ III. $x > 0$

A) I only E) II only I) III only O) I and III only U) insufficient info

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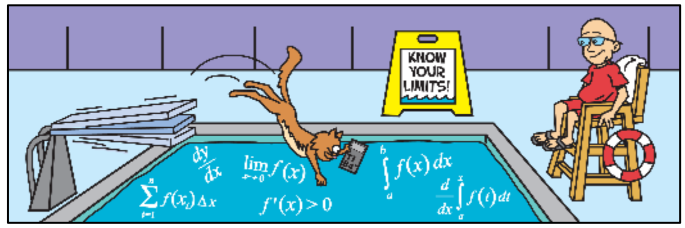


10) Derivatives – ln and exponentials

1. If $f(x)=\ln\left(x-\dfrac{1}{x}\right)$, find $f'(x)$

A) $\dfrac{x^2}{1+x^2}$ E) $\dfrac{x^2+1}{x^3-1}$ I) $\dfrac{x^2+1}{x^3-x}$ O) $\dfrac{x^2}{1-x^2}$ U) $\dfrac{1}{x^3-1}$

2. If $f(x)=\dfrac{\ln x}{e^{2x}}$, use the tangent line approximation of f at $x = 1$ to approximate $f(0.9)$.

J) $\dfrac{1}{10e}$ K) $\dfrac{-1}{10e}$ L) $\dfrac{-1}{9e^2}$ M) $\dfrac{-1}{10e^2}$ N) $\dfrac{1}{10e^2}$

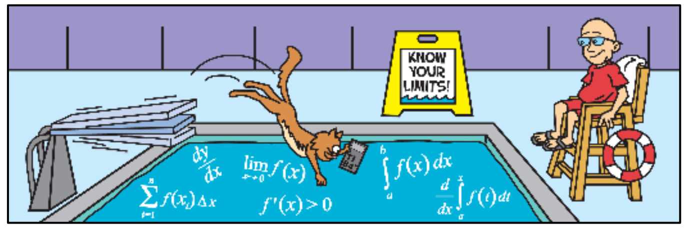
3. If $y=e^{\sin \pi x}$, find $y'(0)+y'(1)+y'(2)+y'(3)+\ldots+y'(100)$

A) 0 E) $-\pi$ I) π O) 1 U) 101π

4. Let f be a function differentiable at all values of x. Values of the function and its derivative $f'(x)$ are given in the table. Let $g(x)=e^{f(-x)}$ and $g'(-1)=12$. Find a.

x	$f(x)$	$f'(x)$
1	a	–3
–1	4	–2

B) ln 6 C) ln 4 D) 4 F) 6 G) ln 12



11) Horizontal and Vertical Tangents

1. If $f'(x)=\ln\left(x^2+1\right)-2$, determine the number of horizontal tangents to the graph of f.

C) 0 D) 1 F) 2 G) 3 H) infinite

2. Let $x^2+xy+y^2=3$. At which points are there horizontal tangents to this curve?

I. (1, –2) II. (2, –1) III. (2, –4)

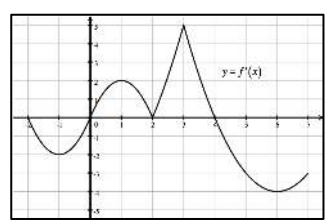
L) I only M) II only N) III only P) I and III only R) I, II, and III

3. Let $2x^3+y^3=6xy$. At which points are there vertical tangents to this curve?

I. (2, –2) II. (2, –4) III. (2, 2)

A) I only E) II only I) III only O) I and III only U) None of these

4. The graph of f', the derivative of f is shown to the right for $-2\le x\le 7$. What are all values of x where f has a horizontal tangent line?



A) $x=-1, 1$ E) $x=-1, 1, 6$ I) $x=-2, 0, 2, 4$ O) $x=-2, 2, 4$ U) $x=-2, 0, 4$

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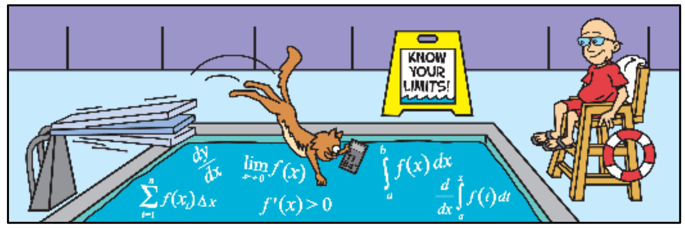
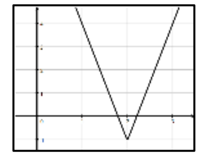


12) Continuity and Differentiability

1. The graph of $f(x) = 5\sqrt{(x-2)^2 + 0.0001} - 1.05$ that passes through the point (2, –1) is shown to the right. Which of the following statements are true?



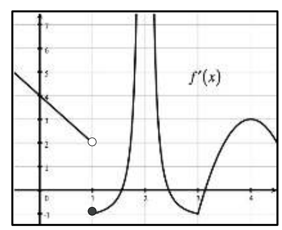
I. f is continuous at $x = 2$. II. f' is continuous at $x = 2$. III. f is differentiable at $x = 2$.

N) none P) I only R) II only S) I and II only T) I, II and III

2. If $f(x) = \begin{cases} e^{ax}, x \le 0 \\ 2(x+a)+b, x > 0 \end{cases}$ and f is differentiable at $x = 0$, find the value of $a + b$.

F) –1 M) 0 S) 1 W) 2 Y) 3

3. The graph of $f'(x)$ is to the right. What is the number of points for which $f(x)$ is **not** continuous on the interval [0, 4]?



C) 0 H) 1 L) 2
N) 3 P) impossible to determine

4. If $f(x) = \begin{cases} 2\sqrt{x+4}, & -4 \le x < 0 \\ 4\cos x + \frac{1}{2}\sin x, & x \ge 0 \end{cases}$

I. $\lim_{x \to 0} f(x)$ exists
II. f is continuous at $x = 0$
III. f' is continuous at $x = 0$
IV. f'' is continuous at $x = 0$

, which of the following is **false**?

D) IV only L) III and IV only P) II, III, and IV only S) I, II, III and IV W) none are false

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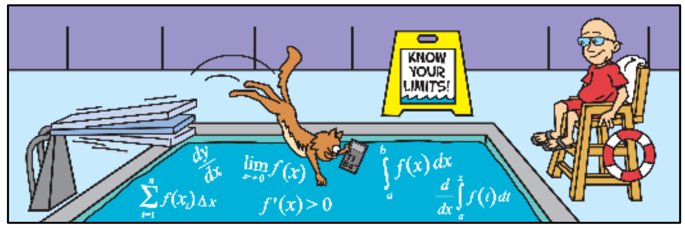
13) Intermediate Value/Rolle's/Mean Value Theorem



1. The functions *f* and *g* are continuous. The continuous function *h* on [1, 4] is given by $h(x) = g(f(x)) + x$. The table below gives values of the functions. What is the minimum number of values of *t* for which $h(t) = 0$?

x	1	2	3	4
$f(x)$	4	1	2	3
$g(x)$	-3	-5	0	6

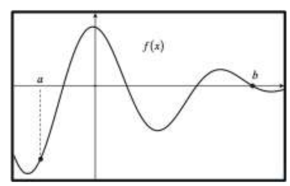
R) 0 S) 1 T) 2 V) 3 W) 4

2. The function *f* is continuous and non-linear for $-4 \le x \le 4$, and $f(-4) = 8$ and $f(4) = -8$. For some *k* where $-4 < k < 4$, which of the following must be true?

I. $f(k) = 0$ II. $f'(k) = 0$ III. $f'(k) = -2$

A) I only E) II only I) III only O) I and II U) I and III

3. The differentiable function *f* on [*a, b*] is shown in the graph to the right. For how many values of *x* on [*a, b*] is the mean value theorem satisfied?



A) 4 E) 3 I) 2 O) 1 U) 0

4. Suppose $f(x)$ is differentiable everywhere and $f(-2) = -5$ and $f'(x) \le 5$ for all values of *x*. Using the Mean-Value Theorem, what is the largest possible value of $f(8)$?

A) 35 E) 45 I) 55 O) 65 U) 75

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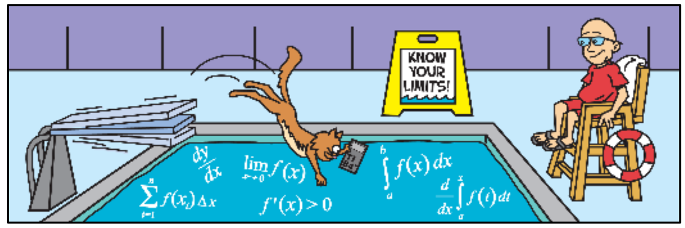
14) Derivative of the Inverse of a Function



1. If $f(x)=x+2^x$ and $g(x)=f^{-1}(x)$, what is the value of $g'(3)$?

M) $\frac{1}{3}$ S) $\frac{1}{1+2\ln 2}$ T) $\frac{1}{1+8\ln 2}$ W) $\frac{1}{8\ln 2}$ Y) $\frac{-(1+8\ln 2)}{121}$

2. The function *f* is differentiable for all real numbers. The table below gives values of the function and its derivatives at $x = -3$ and $x = 2$. If f^{-1} is the inverse function of *f*, find the equation of the tangent line to the graph of $y=f^{-1}(x)$ at $x = -3$.

x	$f(x)$	$f'(x)$
-3	3	4
2	-3	-2

A) $y-2=\frac{-1}{2}(x+3)$ E) $y-2=\frac{1}{4}(x+3)$ I) $y+3=\frac{1}{4}(x-3)$

O) $y+3=\frac{-1}{2}(x-3)$ U) $y-3=-2(x+3)$

3. The graph of $y=\sin^{-1}(x-y)$ passes through the origin. Use the tangent line to the curve at $x = 0$ to approximate the value of *y* at $x=\frac{1}{2}$.

B) $\frac{1}{40}$ C) $\frac{1}{20}$ D) $\frac{1}{4}$ F) $\frac{1}{2}$ G) 1

4. Find the derivative to the inverse to $y=x^2e^x$ at the point $\left(\frac{1}{e},-1\right)$.

P) -1 R) $-e$ S) $\frac{1}{e}$ T) $\frac{-1}{e}$ W) $\frac{-e}{2}$

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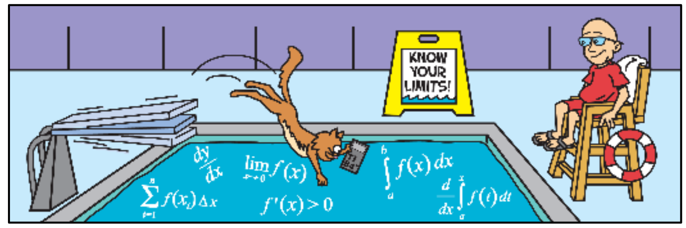
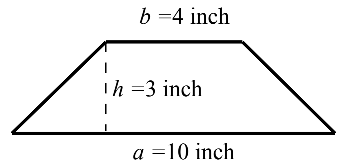


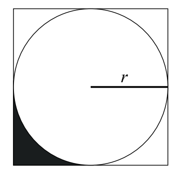
15) Related Rates

1. A trapezoid is pictured on the right. Its bottom base a is not changing while its top base b is increasing at the rate of 3 inches per second while its height h is decreasing at the rate of 0.5 inches per second. How fast is the area of the trapezoid changing, measured in square inches per second, at this moment?



L) –2.5 N) –0.75 T) 1 W) 2 Y) 8

2. A circle of radius r has a square circumscribed about it as shown in the figure to the right. If the radius is increasing at the rate of 2 cm/sec, and the area of the shaded region is increasing at the rate of 8 cm^2/sec, what is the value of r in cm?

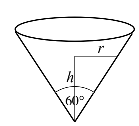


J) $\dfrac{8}{2-\pi}$ K) $\dfrac{16}{2-\pi}$ L) $\dfrac{8}{4-\pi}$ M) $\dfrac{16}{4-\pi}$ N) $\dfrac{32}{4-\pi}$

3. Point A is at (9, 0) and moving towards the origin at 3 units/sec. Point B is at (0, 6) and moving away from the origin at 2 units/sec. One second later, how fast is the distance between the points changing (units/sec)?

B) getting further away at 3.4 F) getting closer at 3.4 R) getting closer at 0.2
T) getting further away at $39/\sqrt{117}$ Z) getting closer at $-15/\sqrt{117}$

4. Water is draining from a conical water tank whose base angle is $60°$ as shown in the figure. When the height of the water is 3 feet, its height is decreasing at 6 inches per hour. How fast is the volume of water decreasing in ft^3/hr?



R) 3π S) $\dfrac{3\pi}{2}$ T) $\dfrac{13\pi}{2}$ V) 18π W) 162π

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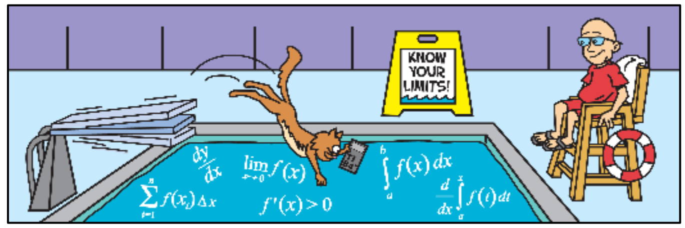
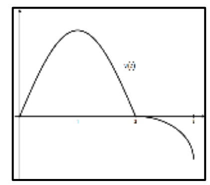


16) Straight-line motion

1. A particle is moving along the x-axis such that at any time $t > 0$, its position is given by $x(t) = (2t^2 - 8t)\ln t - t^2 + 8t$. For what values of t is the particle moving left?

R) (0, 1) S) (0, 2) T) (1, 2) V) (0, ∞) W) (2, ∞)

2. A particle moving along a straight line is moving with velocity $v(t)$ pictured by the graph on the right. For what intervals is the particle speeding up?



I. (0, 1) II. (1, 2) III. (2, 3)

G) I only H) II only J) III only K) I and II only L) I and III only

3. A particle moving along the y-axis has position $y(t)$ measured in feet, velocity $v(t)$ measured in ft/sec, and acceleration $a(t)$ measured in ft/sec^2 at various times t in seconds, given in the chart to the right. Find the average acceleration of the particle on the interval $0 \le t \le 6$.

t	$y(t)$	$v(t)$	$a(t)$
0	–3.5	–5.5	4.5
2	–9.0	–1.1	1.2
4	–9.1	0.9	0.9
6	–7.1	2.3	–0.3

C) 1.30 F) –0.53 K) –0.70 M) –0.80 R) 1.05

4. Using the chart in problem 3, find the average speed of the particle in feet/sec on the interval $0 \le t \le 6$.

A) 4.78 E) 1.63 I) 1.30 O) 0.60 U) 0.53

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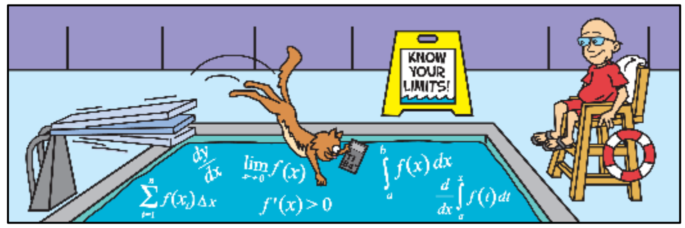
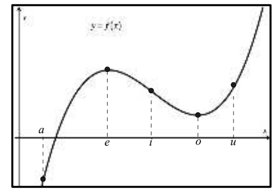
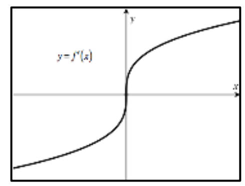
17) Function Analysis



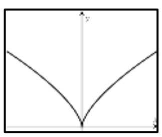
1. The function f is shown in the graph to the right with 5 locations along the x-axis: a, e, i, o, u. Which of the following has the smallest value?

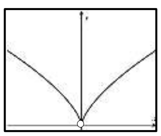


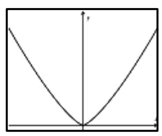
A) $f'(a)$ E) $f''(e)$ I) $f''(i)$ O) $f'(o)$ U) $f''(u)$

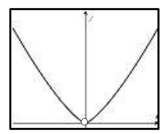


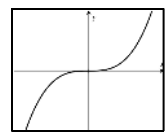
2. The graph of $y = f'(x)$ is shown to the right. Which could be the graph of $f(x)$?


A)


E)


I)


O)


U)

3. If $f'(x) = -(\ln x - 1)^2(\sin x - 2)(x - 3)$, $x > 0$, $f(x)$ has which of the following relative extrema?

I. Relative minimum at $x = e$ II. Relative maximum at $x = 2$ III. Relative minimum at $x = 3$

M) I only N) II only P) III only R) I and III only S) none of these

4. If $f'(x) = 3x^{5/3} - 30x^{2/3}$, for what values of x is f concave up?

B) $(0,4)$ H) $(4,\infty)$ N) $(-\infty,0)\cup(4,\infty)$ T) $(-\infty,0)\cup(2,\infty)$ V) $(0,2)$

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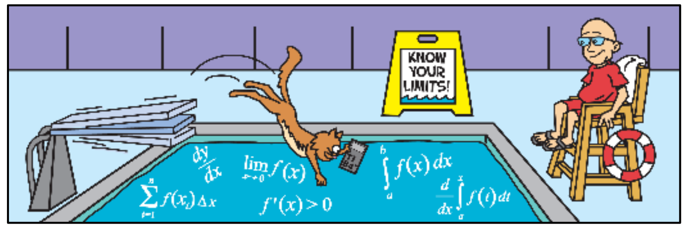


18) Absolute Extrema

1. The absolute maximum value of $y=\left|\sin^2 x+2\sin x\right|$ on $0\le x\le 2\pi$ is

C) $\dfrac{\pi}{2}$ D) 1 F) 3 G) $\dfrac{3\pi}{2}$ H) 2π

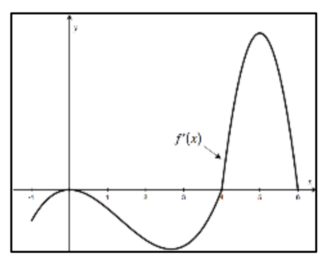
2. For $f(x)=\dfrac{2x+2}{\sqrt{x^2+1}}$, $f'(x)=\dfrac{2-2x}{\left(x^2+1\right)^{3/2}}$. What is the range of $f(x)$?

A) $(-\infty,1]$ E) $(-2,2)$ I) $\left(-2,2\sqrt{2}\right]$ O) $\left[1,2\sqrt{2}\right]$ U) $(-\infty,\infty)$

3. A particle is moving along the x-axis with position function $x(t)=t^4-8t^3+18t^2-216t+1$, $0\le t\le 4$. What is the velocity when the particle achieves minimum acceleration?

S) –12 T) –208 V) –216 W) –240 X) –407

4. Let f be a function that has domain: the closed interval [–1, 6]. Let f have the derivative f' that is continuous with graph of f' shown in the figure to the right. The location of the absolute maximum of f can occur at what value of x?



H) 5 only J) 0 or 5 L) 6 only N) –1 or 6 P) 4 only

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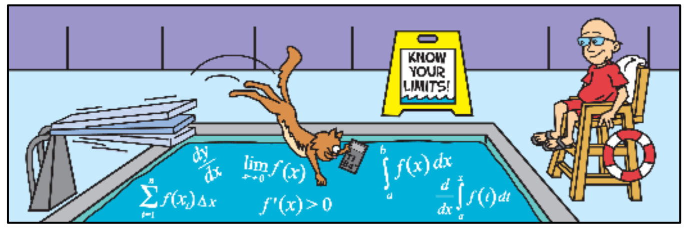

19) 2^{nd} derivative test

1. If $f(x)=k^2\sin x+ke^x$, where k is a nonzero constant, has a critical point at $x = 0$, which of the following statements must be true.

K) f has a relative minimum at $x = k$
L) f has a relative maximum at $x = k$
M) f has a relative minimum at $x = 0$
N) f has a relative maximum at $x = 0$
P) f has a point of inflection at $x = 0$

2. If y is a function such that $\dfrac{dy}{dx}=x^2y-y$, at what point on the graph of y is there a relative maximum?

A) (0, 0) E) (1, 2) I) (–1, 4) O) (2, 1) U) (2, 0)

3. Using the 2^{nd} derivative test to determine functions having a relative extrema at $x = 0$, for which functions would the test be inconclusive?

I. $f(x)=x^2$ II. $f(x)=x^4$ III. $f(x)=2e^x-x^2-2x+1$

A) I only E) II only I) III only O) II and III only U) I, II and III

4. Let f be a function such that $f(4)<0$ and $f'(4)=0$. If $g(x)=e^{-f(x)}$ and g has a relative minimum at $x = 4$, what conclusion can be made about $f''(4)$?

A) $0<f''(4)<1$ E) $f''(4)=0$ I) $f''(4)>0$ O) $f''(4)<0$ U) no conclusion

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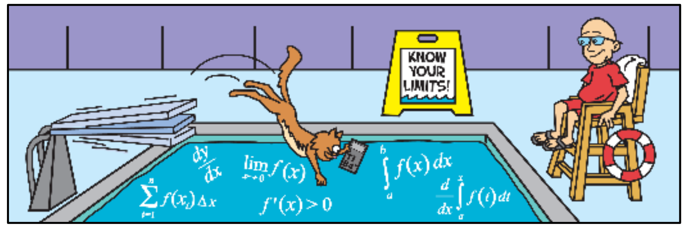
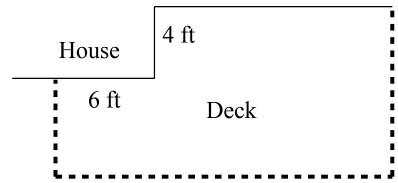


20) Optimization

1. A deck is to be built against a house as shown in the figure to the right. The homeowner has 40 feet of framing material that will be used around the deck (dashed lines). What is the maximum area deck that can be built?



B) 264 ft^2 H) 242 ft^2 M) 240.5 ft^2 T) 218 ft^2 Y) 176 ft^2

2. The speed of traffic through the Holland Tunnel in New York depends on the density of the traffic. Let S be the speed of traffic in miles per hour and D be the density of the traffic in vehicles per mile. If the relationship between S and D is $S = 52 - \frac{D}{3}$ for $D \leq 120$, find the speed that will maximize the hourly flow of traffic, which is measured in vehicles per hour. (Flow = Speed • Density)

A) 12 mph E) 21 mph I) 26 mph O) 42 mph U) 78 mph

3. Find the shortest distance from the graph $y = x^2$ to the point (0, 1).

A) $\frac{1}{2}$ E) $\sqrt{\frac{1}{2}}$ I) $\frac{\sqrt{3}}{2}$ O) 1 U) 2

4. "Life of Pie" sells slices of pizza for \$2.50 and sodas for \$1.40. In a week, they sell 1,000 slices and 500 sodas. The owners calculate that for every 10 cents they increase the price of a slice of pizza, they will sell 20 fewer slices and 10 fewer sodas. What should they price a slice of pizza to maximize their revenue?

R) \$2.75 S) \$3.00 T) \$3.40 V) \$3.60 W) \$4.80

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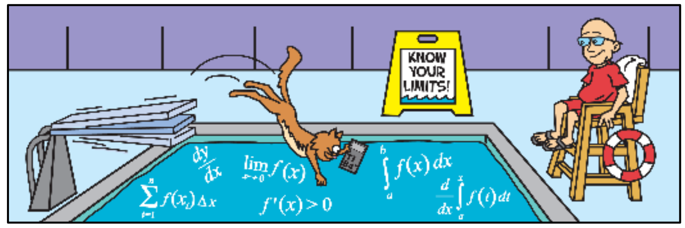

21) Integration

1. $\int (x^3 - 2)^2 dx =$

N) $\dfrac{x^7}{7} + 4x + C$ P) $\dfrac{x^7}{7} - x^4 + 4x + C$ R) $\dfrac{x^7}{7} - x^4 + C$ S) $\dfrac{(x^3-2)^3}{3} + C$ T) $x^6 - 4x^3 + C$

2. $\int \dfrac{\pi}{x^e} dx =$

A) $\dfrac{\pi x^{1-e}}{1-e} + C$ E) $\dfrac{\pi}{(e+1)x^{e+1}} + C$ I) $\dfrac{\pi}{x^{e+1}} + C$ O) $\pi x^{1-e} + C$ U) $\dfrac{\pi x^{e+1}}{e+1} +$

3. $\int \dfrac{3x-4}{x^3} \, dx =$

D) $\dfrac{1}{x^2} + C$ G) $-\dfrac{3}{x} - \dfrac{2}{x^2} + C$ M) $\dfrac{2}{x^2} - \dfrac{3}{x} + C$ R) $\dfrac{1}{x^4} - \dfrac{1}{x^3} + C$ V) $-\dfrac{1}{x^4} - \dfrac{1}{x^3} + C$

4. $\int \dfrac{(\sqrt{x}-1)^2}{\sqrt{x}} \, dx =$

V) $\dfrac{(\sqrt{x}-1)^3}{3\sqrt{x}} + C$ W) $\dfrac{(\sqrt{x}-1)^3}{3} + C$ X) $\dfrac{2}{3}x^{3/2} + 2x^{1/2} + C$

Y) $\dfrac{x^{3/2}}{2} - \dfrac{4x}{3} + x^{1/2} + C$ Z) $\dfrac{2}{3}x^{3/2} - 2x + 2x^{1/2} + C$

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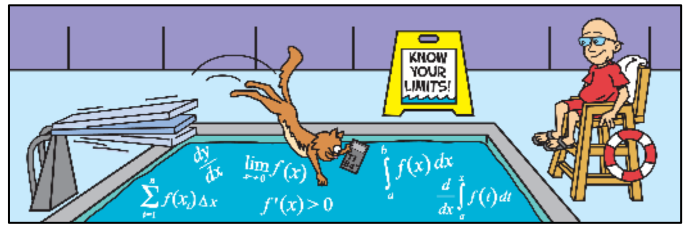


22) Integration with substitution

1. $\int \left(1-\frac{x}{2}\right)^5 dx =$

L) $-12\left(1-\frac{x}{2}\right)^6+C$ R) $\dfrac{-\left(1-\frac{x}{2}\right)^6}{12}+C$ T) $\dfrac{-\left(1-\frac{x}{2}\right)^6}{3}+C$ W) $\dfrac{-\left(1-\frac{x}{2}\right)^6}{6}+C$ Y) $x-\dfrac{x^6}{12}+C$

2. $\int \dfrac{x-3}{\sqrt[3]{x^2-6x+2}}\,dx =$

N) $\frac{3}{4}\left(x^2-6x+2\right)^{2/3}+C$ P) $\frac{3}{8}\left(x^2-6x+2\right)^{4/3}+C$ R) $3\left(x^2-6x+2\right)^{4/3}+C$

S) $3\left(x^2-6x+2\right)^{2/3}+C$ T) $\frac{1}{3}\left(x^2-6x+2\right)^{2/3}+C$

3. $\int f'\left(g(5x)\right)\, g'(5x)\,dx =$

A) $5f(5x)g(5x)+C$ E) $\frac{1}{5}f(5x)g(5x)+C$ I) $f\left(g(5x)\right)+C$

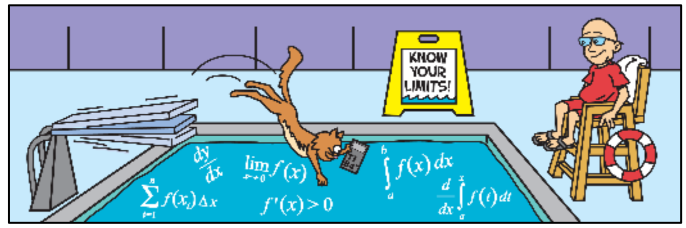
O) $5f\left(g(5x)\right)+C$ U) $\frac{1}{5}f\left(g(5x)\right)+C$

4. $\int \dfrac{x}{\sqrt{x+1}}\,dx =$

A) $2\sqrt{x+1}+C$ E) $\frac{1}{2}\sqrt{x+1}+C$ I) $\dfrac{2(x-2)\sqrt{x+1}}{3}+C$

O) $\frac{2}{3}(x+1)^{3/2}-x+C$ U) $\dfrac{x^2}{2\sqrt{x+1}}+C$

23) Integration with trig functions



1. $\int \sin 7x \cos 7x \, dx =$

I. $\dfrac{\sin^2 7x}{14} + C$ II. $\dfrac{\cos^2 7x}{14} + C$ III. $\dfrac{\sin^2 7x \cos^2 7x}{98} + C$

B) I only C) II only D) III only F) I and II only G) I, II, and III

2. $\int \dfrac{1 - \sin x}{\cos^2 x} \, dx =$

A) $\dfrac{\cos x - 1}{\cos x} + C$ E) $-\sec x + C$ I) $\tan x + \csc x + C$

O) $\tan x - \sec x + C$ U) $-\sec x + \csc x + C$

3. In finding $\int \dfrac{\sin 3x}{\sqrt{\cos 3x}} dx$ by u-substitution, which of the following can be the expression for u?

I. $\cos x$ II. $\cos 3x$ III. $\sqrt{\cos 3x}$

P) I only R) II only S) III only T) II and III only W) I, II, and III

4. $\int \sec^4 4x \cdot \tan 4x \, dx =$

A) $\dfrac{\tan^2 4x \sec^2 4x}{8} + C$ E) $4\sec^4 4x + C$ I) $\dfrac{5\sec^5 4x}{4} + C$ O) $\dfrac{\sec^5 4x}{20} + C$ U) $\dfrac{\sec^4 4x}{16} + C$

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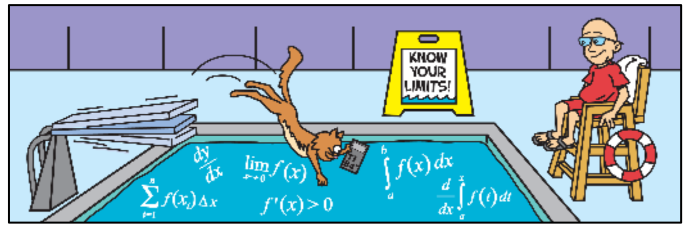
24) Integration with ln and e



1. $\int \frac{1+\ln x}{x}\,dx =$

A) $\ln\left|x+\ln x\right|+C$ E) $\ln|x|+\ln\left|\ln x\right|+C$ I) $\frac{(1+\ln x)^2}{2}+C$ O) $\frac{-x+(\ln x)^2}{2}+C$ U) $x+\frac{(\ln x)^2}{2}+C$

2. $\int \frac{1}{\cos^2 2\theta(1+\tan 2\theta)}\,d\theta =$

D) $\frac{1}{2}\ln\left|\tan 2\theta\right|+C$ J) $\frac{2}{(1+\tan 2\theta)^2}+C$ L) $\frac{1}{2(1+\tan 2\theta)^2}+C$

P) $2\ln\left|1+\tan 2\theta\right|+C$ S) $\frac{1}{2}\ln\left|1+\tan 2\theta\right|+C$

3. Find $\int \frac{\sqrt{a+e^{-x}}}{e^x}\,dx$ where a is a constant.

R) $\left(a+e^{-x}\right)^{3/2}+C$ T) $-\frac{2}{3}\left(a+e^{-x}\right)^{3/2}+C$ V) $-\frac{2}{3}\ln\left|a+e^{-x}\right|+C$

W) $\ln\left|a+e^{-x}\right|+C$ Z) $-\ln\left|a+e^{-x}\right|+C$

4. $\frac{1}{2}\int xe^{x^2}\tan e^{x^2}\,dx =$

D) $\ln\left|\cos e^{x^2}\right|+C$ G) $-\ln\left|\cos e^{x^2}\right|+C$ K) $\frac{1}{4}\ln\left|\cos e^{x^2}\right|+C$ S) $-\frac{1}{4}\ln\left|\cos e^{x^2}\right|+C$ Y) $\sec^2\left(e^{x^2}\right)+C$

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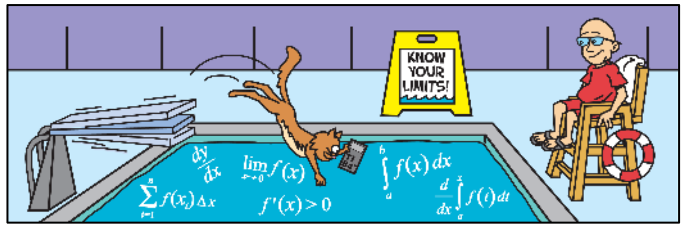
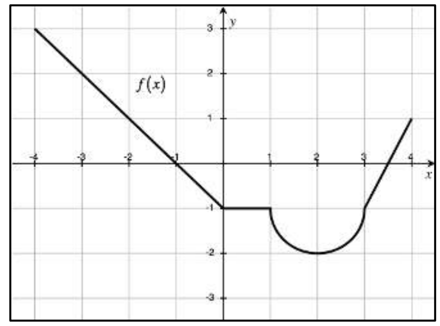


25) Definite Integral as Area

For problems 1 – 3, below, the graph of $f(x)$, made up of straight lines and a semicircle, is denoted on the figure to the right and $F(x)=\int_1^x f(t)\,dt$.



1. Find $F(4)$.

   A) $-4-\frac{\pi}{2}$ E) $-3-\frac{\pi}{2}$ I) $-3-\pi$

   O) $-2-\pi$ U) $-2-\frac{\pi}{2}$

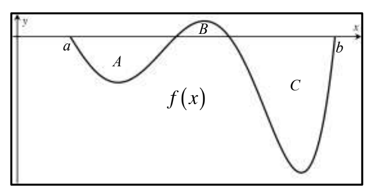
2. On the interval [–4, 4], at how many locations does $F(x)=0$?

   P) 0 Q) 1 R) 2 S) 3 T) 4

3. Find $\int_0^3 \left[f(x-2)+2\right]dx$

   J) –1 K) $-2+\frac{\pi}{2}$ L) $3+\frac{\pi}{2}$ M) 5 N) 7

4. If $f(x)$ is shown in the graph to the right drawn to scale, with A, B, and C representing positive areas between $f(x)$ and the x-axis, find the value of $\int_a^b f(x)\,dx+\left|\int_a^b f(x)\,dx\right|+\int_a^b \left|f(x)\right|dx$.



   B) $-A+B-C$ F) $3A+3B+3C$ N) $A+2B+C$ T) $A+3B+C$ Z) $A+B+C$

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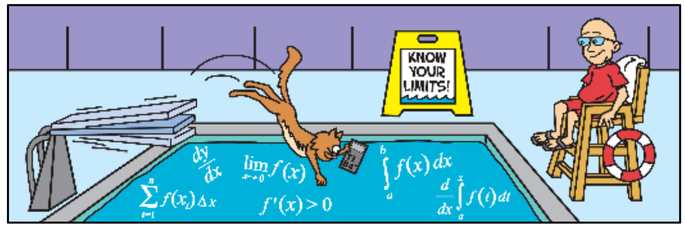


26) Riemann Sums/Trapezoidal Rule

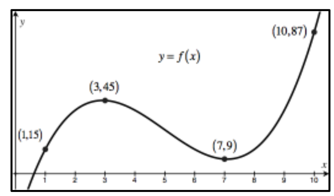
1. Approximations to $\int_0^3 e^x dx$ are made using left Riemann sums (L), right Riemann sums (R), and trapezoids (T), each using 3 subintervals of equal length. Arrange the calculations from smallest to largest.

A) L-T-R E) L-R-T I) R-T-L O) R-L-T U) T-L-R

2. The expression $\frac{1}{25}\left(2^{50/25} + 2^{51/25} + 2^{52/25} + \ \ldots\ + 2^{99/25}\right)$ is a Riemann sum approximation for

A) $\int_{50}^{99} 2^x\,dx$ E) $\frac{1}{25}\int_{50}^{99} 2^x\,dx$ I) $\int_{2}^{4} 2^x\,dx$ O) $\frac{1}{25}\int_{2}^{4} 2^x\,dx$ U) $\frac{1}{25}\int_{2}^{99/25} 2^x\,dx$

3. The graph of $y = f(x)$ passes through the points (1, 15) and (10, 87) and has a relative maximum at (3, 45) and a relative minimum at (7, 9) as shown in the figure to the right. If $\int_1^{10} f(x)\,dx$ is to be computed with 9 equal subintervals, what is the difference between the right and left Riemann sum calculations?



P) 4 R) 8 S) 36 T) 72 V) insufficient info

4. The function *f* is continuous on the closed interval [0, 6] and has the values given in the table. The trapezoidal approximation for $\int_0^6 f(x)\,dx$, found with 3 subintervals, is $15k$. What is the largest trapezoidal approximation satisfying this relationship?

x	0	1	4	6
$f(x)$	2	k^2	$2k$	11

A) 2 E) 3 I) 30 O) 45 U) 60

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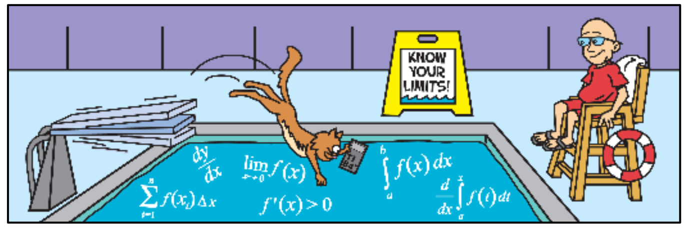

27) Fundamental Theorem of Calculus

1. $\displaystyle\int_0^{\pi/4} \frac{e^{2\tan x}}{\cos^2 x}\,dx =$

M) $2e^2$ N) $2(e-1)$ P) $\frac{1}{2}(e-1)$ R) $\frac{e^2}{2}$ S) $\frac{1}{2}(e^2-1)$

2. $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt[3]{10-6x}}\,dx =$

A) $6\displaystyle\int_4^{16} u^{-1/3}du$ E) $\frac{1}{6}\displaystyle\int_4^{16} u^{-1/3}du$ I) $\frac{-1}{6}\displaystyle\int_4^{16} u^{-1/3}du$ O) $\frac{-1}{6}\displaystyle\int_{-1}^{1} u^{-1/3}du$ U) $6\displaystyle\int_{-1}^{1} u^{-1/3}du$

3. (Calculator Active) Let $F(x)$ be an antiderivative of $\sin(x^2+x+1)$. If $F(\pi)=-\pi$, then $F(0)=$

P) –3.636 R) 2.647 S) 0.494 T) –0.494 W) –2.647

4. If f is a continuous function and $F'(x)=f(x)$ for all real numbers $x \geq 0$, then $\displaystyle\int_0^2 \frac{f\sqrt{4x+1}}{\sqrt{4x+1}}\,dx =$

C) $2[F(3)-F(1)]$ D) $\frac{1}{2}[F(3)-F(1)]$ F) $2[F(2)-F(0)]$ G) $\frac{1}{2}[F(2)-F(0)]$ H) $F(2)-F(0)$

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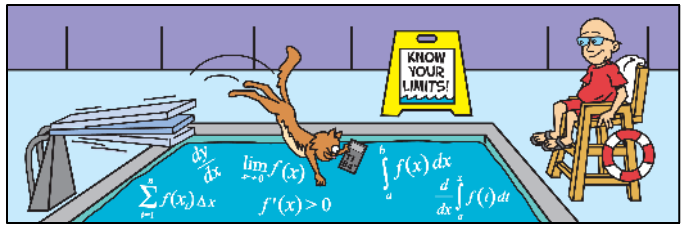
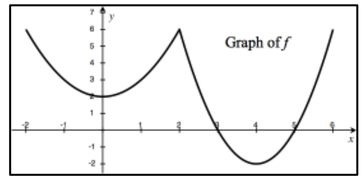


28) The accumulation function



1. Let f be the function in the graph to the right on the domain [–2, 6]. If $F(x)=\int_0^x f(t)\,dt$, in what interval(s) is F decreasing?

A) $(-2,0)\cup(2,3)$ E) $(-2,0)\cup(2,4)$ I) $(3,5)$ only O) $(3,4)$ only U) no intervals

2. Using the same graph of f as in problem 1, in what intervals is F concave down?

A) $(-2,0)\cup(2,4)$ E) $(-2,0)\cup(2,3)$ I) $(3,4)$ only O) $(3,5)$ only U) no intervals

3. Let $f(x)$ be a function continuous on the interval [–3,3). The function is differentiable except at x = 1. The function f and its derivative have the properties in the table below, where DNE means that the derivative doesn't exist. If $g(x)=\int_{-1}^{x} f(t)\,dt$, for what values of x does g have a relative maximum?

x	-3	$-3<x<-1$	-1	$-1<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	-4	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	5	Positive	0	Positive	DNE	Negative	0	Negative

I. $x=-1$ II. $x=1$ III. $x=2$

C) I only H) II only M) III only R) I and II only W) None

4. Let $f(x)=x^2$ and $g(x)=\sin x$. If $h(x)=\int_{-1}^{g(x)} f(t)\,dt$, find the value of $h'(\pi/6)$.

B) $\frac{1}{4}$ C) $\frac{\sqrt{3}}{8}$ D) $\frac{\pi\sqrt{3}}{6}$ F) $\left(\frac{\pi}{3}+2\right)\left(\frac{\sqrt{3}}{2}\right)$ G) $\sqrt{3}$

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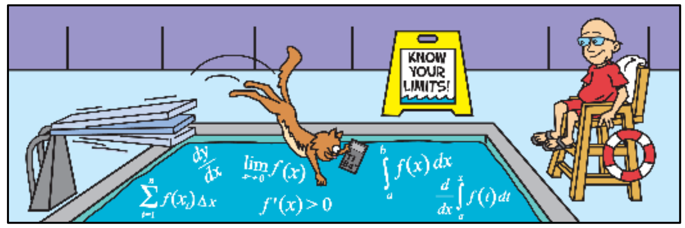
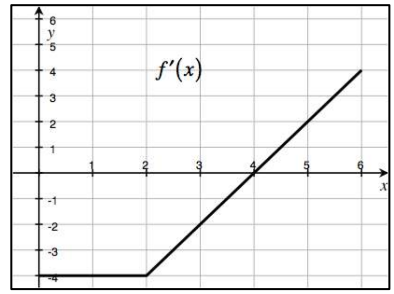
29) Integral of rate of change





1. The graph of $f'(x)$, the derivative of f, is shown in the figure to the right. If $f(0)=3$, then $f(5)=?$

   A) -14 E) 2 I) 5
   O) -11 U) -8

2 (Calculator Active) A pool contains 5,000 gallons of water. When the pool has a lot of kids, there is a lot of splashing and water leaves the pool. Water occasionally is also pumped into the pool. Over a 5 hour period, the rate that the pool level changes is given by $r(t)=-12+6t+7t^2-2t^3$, measured in gallons per hour. Which of the following gives the change in water level in gallons when water in the pool is rising?

   A) $\int_{1.071}^{3.875} r(t)\,dt$ E) $5000-\int_{1.071}^{3.875} r(t)\,dt$ I) $\int_{1.071}^{3.875} r'(t)\,dt$ O) $\int_{0}^{2.703} r(t)\,dt$ U) $5000-\int_{0}^{2.703} r(t)\,dt$

3. The table to the right gives values of a function f and its derivative at selected values of x. If $f'(x)$ is continuous on the interval [-4, 4], what is the value of $\int_{-2}^{2} f'\left(\frac{x}{2}\right)dx$?

x	-4	-2	-1	1	2	4
$f(x)$	12	4	3	-2	4	10
$f'(x)$	-7	-3	-1	3	5	9

   B) 16 D) 8 G) 0 J) -5 L) -10

4. (Calculator Active) To make a ring, a metal is melted to $1500°$ F in a form and then exposed to the outside air at $72°$ F. The metal cools at the rate of $180e^{-0.1t}$ degrees Fahrenheit per second. To the nearest degree, what is the temperature of the metal 15 seconds after it is taken from the heat?

   S) 40° T) 102° V) 174° W) 1398° Y) 1460°

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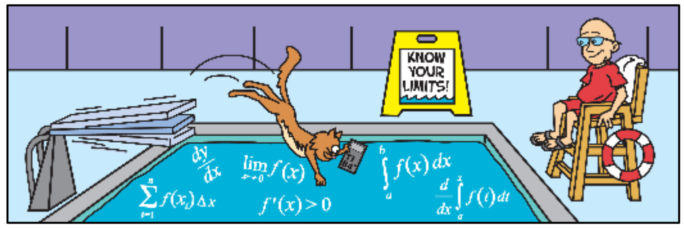
30) Average value of a function



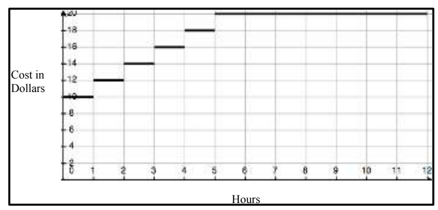
1. The velocity of a particle moving along the x-axis is given by $v(t)=\dfrac{\ln t}{t}$. What is the average velocity of the particle from $t = 1$ to $t = e$?

A) $\dfrac{1}{e^2-e}$ E) $\dfrac{1}{2}$ I) $\dfrac{e^{2e}-1}{2}$ O) $\dfrac{e^{2e}}{2}$ U) $\dfrac{1}{2e-2}$

2. Find the average value of $(1+\sin x)(1+\cos x)$ on [π, 2π].

M) 1 N) $\dfrac{\pi-2}{\pi}$ P) $\dfrac{\pi+2}{\pi}$ R) 3π S) $\pi-2$

3. A parking lot in a busy shopping area charges for parking. It costs \$10 for the first hour or part and an extra two dollars for each additional hour or part the shopper uses the lot. There is a maximum of \$20 charged and a maximum of 12 hours. The graph of the parking price for the number of hours is shown in the figure to the right. What is the average price of parking over a 12-hour period?



A) \$15 E) \$17 I) \$17.33 O) \$17.50 U) 18

4. Let $f(x)=\dfrac{1}{x}$. For how many value of $b > 1$ does the average value of $f(x)$ on the interval [1, b] equal the average rate of change of f on [1, b] ?

C) 0 F) 1 K) 2 R) infinite T) depends on b

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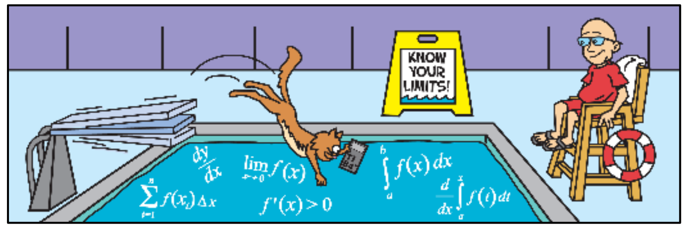


31) Straight-line motion - integrals

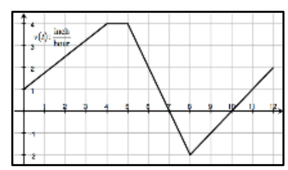
1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = \dfrac{t}{t^2 + e}$. If the position of the particle at $t = 0$ is 1, find the position of the particle at $t = \sqrt{e}$.

A) $\dfrac{\ln 2}{2}$ E) $\dfrac{\ln 2}{2} + 1$ I) $\dfrac{\ln 2}{2} + \dfrac{1}{2}$ O) $\dfrac{\ln 2}{2} + \dfrac{3}{2}$ U) $\dfrac{\ln 2}{2} - \dfrac{1}{2}$

2. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t^2 - t$. Find the distance the particle travels from $t = 0$ to $t = 3$.

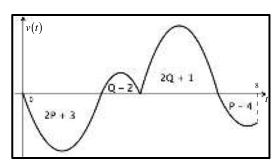
K) $\dfrac{9}{2}$ L) $\dfrac{29}{6}$ M) 6 N) $\dfrac{3}{2}$ P) $\dfrac{7}{2}$

3. An outdoor display vertical thermometer is measured in inches with 2 degrees Fahrenheit per inch. At 8AM, the temperature is $20°$ F. A graph of the velocity of the temperature bar in inches per hour over 12 hours is shown to the right. What is the temperature at 8 PM?



D) 34° G) 37° K) 43° N) 54° R) 66°

4. A particle moves along the x- axis with velocity $v(t)$ for $0 \leq t \leq 8$. The graph of $v(t)$ is shown to the right with the positive areas as functions of P and Q. What is the difference between the distance the particle travels and its displacement for $0 < t \leq 8$?



S) 2 T) 6P – 2 V) 6P – 6Q – 2 W) 6P + 6Q – 2 Y) 6Q – 2

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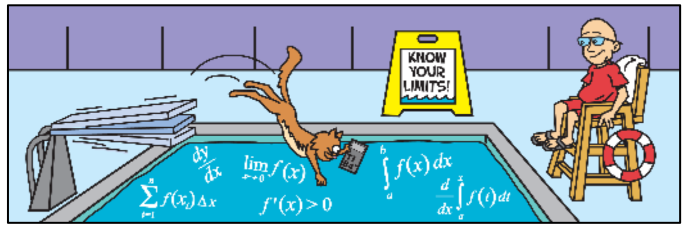

32) Calculus & inverse trig

1. If $f(x)=\sin\left(\cos^{-1}x\right)$, find $f'(x)$.

A) $\dfrac{x}{\sqrt{1-x^2}}$ E) $\dfrac{-x}{\sqrt{1-x^2}}$ I) $\dfrac{-1}{\sqrt{1-x^2}}$ O) $\dfrac{-1}{1+x^2}$ U) $\dfrac{x}{1+x^2}$

2. Find the equation of the tangent line to $f(x)=\tan^{-1}(\ln x)$ at $x=e$.

B) $y-\dfrac{\pi}{4}=\dfrac{1}{2}(x-e)$ C) $y-\dfrac{\pi}{4}=\dfrac{1}{2e}(x-e)$ D) $y=\dfrac{1}{2}(x-e)$

F) $y=\dfrac{1}{2e}(x-e)$ G) $y-\dfrac{\pi}{2}=\dfrac{1}{2e}(x-e)$

3. $\displaystyle\int\frac{e^x}{2+e^{2x}}\,dx=$

A) $\tan^{-1}\left(\dfrac{e^x}{2}\right)+C$ E) $\tan^{-1}\left(\dfrac{\sqrt{2}e^x}{2}\right)+C$ I) $\dfrac{\sqrt{2}\tan^{-1}\left(\dfrac{\sqrt{2}e^x}{2}\right)}{2}+C$

O) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{e^x}{2}\right)+C$ U) $\dfrac{1}{2}\ln\left|2+e^{2x}\right|+C$

4. Find the value of $\displaystyle\int_{-1}^{1}\frac{1}{\sqrt{4-x^2}}\,dx$.

P) 0 R) $\dfrac{\pi}{6}$ S) $\dfrac{\pi}{3}$ T) π W) 2π

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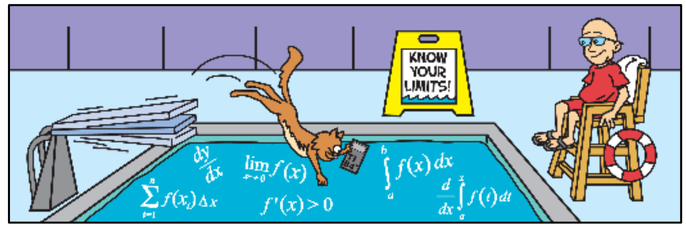
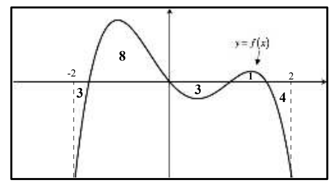


33) Area

1. What is the area of the region enclosed by $y = 1 + 2x - 3x^2$ and $y = -x - 5$?

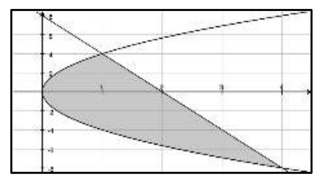
B) $\frac{9}{2}$ D) $\frac{27}{2}$ F) $\frac{9}{4}$ H) $\frac{33}{2}$ J) $\frac{37}{2}$

2. The region between $y = f(x)$ and the x-axis between $x = -2$ and $x = 2$ is shown in the graph to the right. Find the value of $\int_{-2}^{2} \left[|x| - f(x) \right] dx$.



A) 5 E) 3 I) –1
O) –15 U) –19

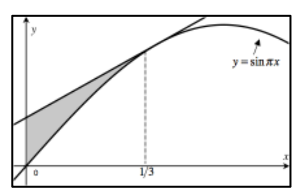
3. The shaded region, enclosed by the graphs of $x = \frac{y^2}{16}$ and $y = 8 - 4x$ is shown in the figure to the right. Which of the following calculations accurately computes the area of this region?



I. $\int_0^4 \left[4\sqrt{x} - (8 - 4x) \right] dx$ II. $\int_0^1 8\sqrt{x}\, dx + \int_1^4 \left(8 - 4x + 4\sqrt{x} \right) dx$ III. $\int_{-8}^{4} \left(\frac{8 - x}{4} - \frac{x^2}{16} \right) dx$

C) I only F) II only K) III only M) II and III only T) I, II, and III

4. (Calculator Active) On the graph to the right, the line is tangent to the graph of $y = \sin \pi x$ at $x = 1/3$. Find the area of the shaded region.



P) 0.042 R) 0.102 S) 0.217
T) 0.361 V) 0.420

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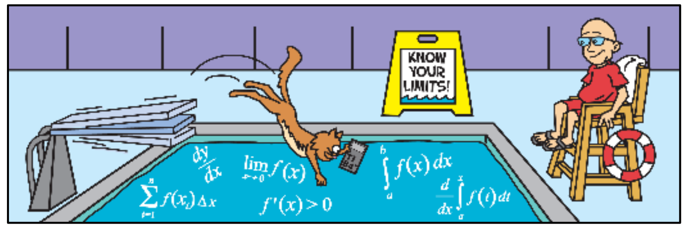
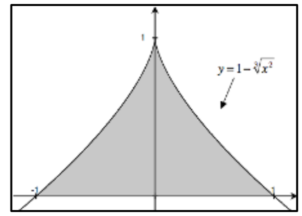


34) Volume

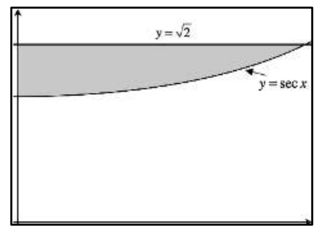
1. The region bounded the function $y=1-\sqrt[3]{x^2}$ and the x-axis as shown in the figure to the right is rotated about the x-axis. Find the volume of rotation.



A) $\frac{8\pi}{35}$ E) $\frac{16\pi}{35}$ I) $\frac{2\pi}{5}$

O) $\frac{4\pi}{5}$ U) $\frac{20\pi}{7}$

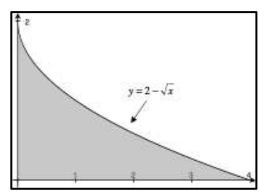
2. The region bounded by the curves $y=\sec x, y=\sqrt{2}$, and the y-axis is shaded in the figure to the right. If this region is rotated about the line $y=\sqrt{2}$, which of the following represents the volume of the solid?



A) $\pi\int_0^{\pi/4}\left(\sqrt{2}-\sec x\right)^2 dx$ E) $\pi\int_0^{\pi/4}\left(2-\sec^2 x\right) dx$

I) $\pi\int_0^{\pi/4}\left(2-\sec^2 x\right)^2 dx$ O) $\pi\int_0^{\pi/4}\left(\sqrt{2}-\sec x\right) dx$ U) $\pi\int_0^{\pi/4}\left(\sqrt{2}-\sec^2 x\right) dx$

3. As shown in the figure to the right, the region bounded by the curve $y=2-\sqrt{x}$ and the x- and y-axes are rotated about different lines below. Arrange the volumes of the resulting solids from largest to smallest.



I. the x-axis II. the y-axis III. the line $y = 2$

A) III – II – I E) II – III – I I) I – II – III O) II – I – III U) III – I – II

4. Let R be the region bounded by the graphs of $y=e^x, y=-1, x=1,$ and the y-axis. R is the base of a solid with cross sections perpendicular to the x-axis as squares. Find the volume of the solid.

K) $\frac{e^2}{2}+2e+1$ L) $\frac{e^2}{2}+2e-\frac{3}{2}$ M) $\frac{e^2}{2}-2e+\frac{5}{2}$ N) $\frac{e^2}{2}-2e+1$ P) $\frac{e^2}{2}+2e+\frac{5}{2}$

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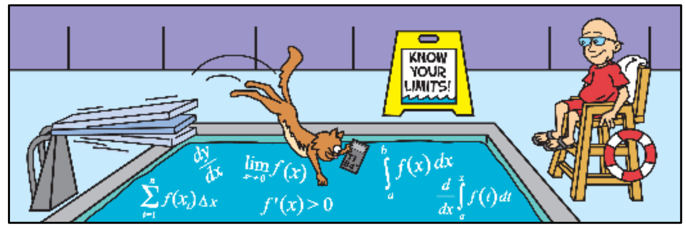
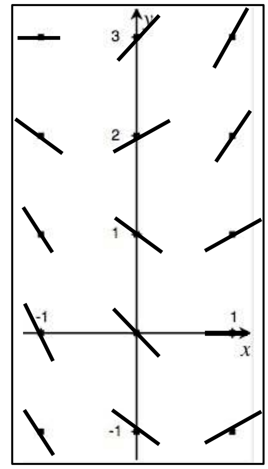


35) Slope fields / differential equations

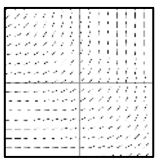
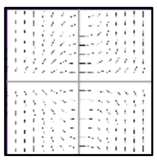
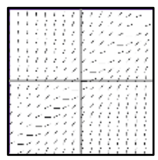
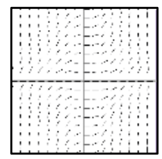
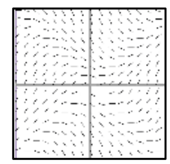
1. The slope field in the figure to the right could be generated by the differential equation:



P) $\frac{dy}{dx}=2\cos(\pi x)+2$ Q) $\frac{dy}{dx}=x+\sin\left(\frac{\pi y}{2}\right)$ R) $\frac{dy}{dx}=x-\cos\left(\frac{\pi y}{3}\right)$

S) $\frac{dy}{dx}=y+\cos\left(\frac{\pi x}{3}\right)$ T) $\frac{dy}{dx}=x-\cos\left(\frac{\pi y}{2}\right)$

2. The slope field for $\frac{dy}{dx}=\left|x^2 y\right|$ is which of the following?

F)  H)  M)  S)  W) 

3. If $\frac{dy}{dx}=3\sqrt{xy}$, and if $y = 4$ when $x = 1$, then when $x = 2$, $y =$

A) 8 E) $9+4\sqrt{2}$ I) $9-4\sqrt{2}$ O) $9-2\sqrt{2}$ U) 9

4. For the function passing through the point (π, 1), defined by the differential equation $\frac{dy}{dx}=y\sin x$, find the range of the function.

B) $\left[\frac{1}{e^2},1\right]$ C) $\left[\frac{1}{e^2},0\right]$ D) $\left[0,e^2\right]$ F) $\left[1,e^2\right]$ G) $\left[\frac{1}{e},e\right]$