Regina Summer Math Review

For students who will be taking

Pre-Calculus

Completed review packet due the first day of classes

Summer Review for Pre-Calculus

This is a list of topics that you should know when you take Pre-Calculus at Regina. We have created this packet to help students better prepare for this challenging course. The better you know these concepts, the easier the transition to Pre-Calculus will be.

A) The things you want to know well when you come in the door are first and are **boldfaced.**

B) Things you should have good familiarity with come second and are <u>underlined.</u>

C) The topics in parenthesis are things that you should have been introduced to and we may cover further in Pre-Calculus.

THE TOPICS

Manipulative Algebra: You should know all the following techniques:

1. **Good use of the distributive property, in exponential, radical, and linear situations**
2. **Gathering like terms, simplifying correctly, order of operations**
3. **How to simplify a binomial and how NOT to**
4. **How to simplify a radical and how NOT to**
5. **How to work with imaginary and radical answers**
6. **Basic factoring and the zero product property, including the difference of 2 squares**
7. **Quadratic formula**
8. **Exponents: all properties, including fractional exponents and their link to radicals**
9. **How to solve inequalities and compound inequalities (AND/OR)**
10. **Fractions: how to +-*/, use LCD's, simplify (cancel), and solve fractional equations**
11. <u>Polynomial long division</u>
12. (How to solve absolute value inequalities)

Functions:

13. **Know the formal definition of a function**
14. **Use function notation to find a function value**
15. **Understand the implications of function notation (ex: what does $f(2) = 3$ mean?)**
16. <u>Graphs of power, polynomial, and rational functions (this is sometimes bypassed in HAA)</u>
17. <u>Basic combined functions (+-*/), composite functions, and inverse functions</u>

Linear Functions:

18. **Point-slope form of a line**
19. **Slope-intercept form of a line**
20. **How to find slope and what it means**
21. **How to find an equation from information about a graph**
22. **How to graph a given equation, even if it's not in the right form**

23. **How to find an intersection using graphing, substitution, and adding equations**
24. **How to graph a linear inequality and the solutions for a system of linear inequalities**
25. **Parallel and perpendicular lines**

Quadratic Functions:

26. **Be able to graph quadratics from the following forms: vertex and root**
27. **Understand the effects of a and c (standard form)**
28. **Understand the effects of a, h, and k (vertex form)**
29. **Understand the effects of a and the roots (root form)**
30. **Find the x-intercepts of a parabola in any form, using appropriate algebra**
31. **Be able to find an equation given sufficient graphical information**
32. Complete the square to graph a parabola

Calculator Use:

33. **Be proficient with the use of ANS, ENTRY, INS, DEL**
34. **Be able to graph in function mode**
35. **Understand the implications of the WINDOW (including distortion), and be proficient in altering the window to get a good graph**
36. **Use Trace and basic Zooms**
37. Use the STO button to store variables in the calculator
38. Use of linear regression to find the line of best fit

Exponential and Logarithmic Functions:

39. Know the basic form of exponential functions and the role each variable has in basic situations
40. Solve exponential equations using logarithms
41. Be able to sketch exponential graphs
42. From an exponential graph, sketch basic log graphs
43. Know all the logarithmic properties

Odds and Ends:

44. Intro to the number e through "compounded continuously"
45. Variation: Inverse and direct with various powers of the independent variables
46. (Matrices: +-*, using them to solve linear equations through inverses)
47. (Probability: Basic concept, compound events (both and and or), and conditional probability)
48. (Combinatorics: Combinations, permutations, the binomial theorem, and Pascal's triangle.)

Algebra 2 – Things to Remember!

Exponents:

$x^0 = 1$ $\qquad x^{-m} = \frac{1}{x^m}$

$x^m \bullet x^n = x^{m+n}$ $\qquad (x^n)^m = x^{n \bullet m}$

$\frac{x^m}{x^n} = x^{m-n}$ $\qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$(xy)^n = x^n \bullet y^n$

Complex Numbers:

$\sqrt{-1} = i$ $\qquad \sqrt{-a} = i\sqrt{a}; a \geq 0$

$i^2 = -1$ $\qquad i^{14} = i^2 = -1$ divide exponent by 4, use remainder, solve.

$(a+bi)$ conjugate $(a-bi)$

$(a+bi)(a-bi) = a^2 + b^2$

$|a+bi| = \sqrt{a^2+b^2}$ absolute value=magnitude

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\ln x = \log_e x$ natural log

$e = 2.71828\ldots$

$\log x = \log_{10} x$ common log

Change of base formula:

$\log_b a = \frac{\log a}{\log b}$

Properties of Logs:

$\log_b b = 1$ $\qquad \log_b 1 = 0$

$\log_b (m \bullet n) = \log_b m + \log_b n$

$\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b (m^r) = r \log_b m$

Domain: $\log_b x$ is $x > 0$

Factoring:

Look to see if there is a GCF (greatest common factor) first. $ab + ac = a(b+c)$

$x^2 - a^2 = (x-a)(x+a)$

$(x+a)^2 = x^2 + 2ax + a^2$

$(x-a)^2 = x^2 - 2ax + a^2$

Factor by Grouping:

$x^3 + 2x^2 - 3x - 6$

$(x^3 + 2x^2) - (3x+6)$ group

$x^2(x+2) - 3(x+2)$ factor each

$(x^2 - 3)(x+2)$ factor

Exponentials $e^x = \exp(x)$

$b^x = b^y \rightarrow x = y$ $(b > 0$ and $b \neq 1)$

If the bases are the same, set the exponents equal and solve.

Solving exponential equations:

1. Isolate exponential expression.
2. Take *log* or *ln* of both sides.
3. Solve for the variable.

$\ln(x)$ and e^x are inverse functions

$\ln e^x = x$ $\qquad e^{\ln x} = x$

$\ln e = 1$ $\qquad e^{\ln 4} = 4$

$e^{2\ln 3} = e^{\ln 3^2} = 9$

Quadratic Equations: $ax^2 + bx + c = 0$ (Set = 0.)

Solve by factoring, completing the square, quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac > 0$ two real unequal roots

$b^2 - 4ac = 0$ repeated real roots

$b^2 - 4ac < 0$ two complex roots

Square root property: If $x^2 = m$, then $x = \pm\sqrt{m}$

Completing the square: $x^2 - 2x - 5 = 0$

1. If other than one, divide by coefficient of x^2
2. Move constant term to other side $x^2 - 2x = 5$
3. Take half of coefficient of x, square it, add to both sides

$$x^2 - 2x + \boxed{1} = 5 + \boxed{1}$$

4. Factor perfect square on left side. $(x-1)^2 = 6$
5. Use square root property to solve and get two answers. $x = 1 \pm \sqrt{6}$

Sum of roots: $r_1 + r_2 = -\frac{b}{a}$ **Product of roots:** $r_1 \bullet r_2 = \frac{c}{a}$

Variation: always involves the constant of proportionality, k. Find k, and then proceed.

Direct variation: $y = kx$

Inverse variation: $y = \frac{k}{x}$

Varies jointly: $y = kxj$

Combo: Sales vary directly with advertising and inversely with candy cost. $y = \frac{ka}{c}$

Absolute Value: $|a| > 0$

$$|a| = \begin{cases} a; & a \geq 0 \\ -a; & a < 0 \end{cases}$$

$|m| = b \Rightarrow m = -b$ or $m = b$

$|m| < b \Rightarrow -b < m < b$

$|m| > b \Rightarrow m > b$ or $m < -b$

Inequalities: $x^2 + x - 12 \leq 0$ Change to =, factor, locate critical points on number line, check each section.

$(x+4)(x-3) = 0$

$x = -4;\ x = 3$

ANSWER: $-4 \leq x \leq 3$ or $[-4, 3]$ (in interval notation)

Radicals: Remember to use fractional exponents.

$\sqrt[a]{x} = x^{\frac{1}{a}}$ $\qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\sqrt[n]{a^n} = a$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\cdot\sqrt[n]{b}$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify: look for perfect powers.

$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$

$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8\cdot 9x^9y^6y^2z^3} = 2x^3y^2z\sqrt[3]{9y^2}$

Use conjugates to rationalize denominators:

$$\frac{5}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10-5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}} = 10-5\sqrt{3}$$

Equations: isolate the radical; square both sides to eliminate radical; combine; solve.

$2x-5\sqrt{x}-3=0 \quad \rightarrow \quad (2x-3)^2 = (5\sqrt{x})^2$

$4x^2-12x+9 = 25x \quad \rightarrow \quad solve: x=9;\ x=1/4$

CHECK ANSWERS. Answer only $x=9$.

Functions: A function is a set of ordered pairs in which each x-element has only ONE y-element associated with it.

Vertical Line Test: is this graph a function?
Domain: x-values used; **Range:** y-values used
Onto: all elements in B used.
1-to-1: no element in B used more than once.
Composition: $(f\circ g)(x) = f(g(x))$
Inverse functions f & g: $f(g(x)) = g(f(x)) = x$
Horizontal line test: will inverse be a function?

Transformations:
$-f(x)$ over x-axis; $f(-x)$ over y-axis
$f(x+a)$ horizontal shift; $f(x)+a$ vertical shift
$f(ax)$ stretch horizontal; $af(x)$ stretch vertical

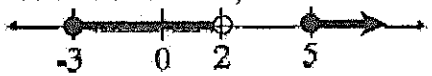
Working with Rationals (Fractions):
Simplify:
remember to look for a factoring of -1:

$$\frac{3x-1}{1-3x} = \frac{-1(-3x+1)}{1-3x} = -1$$

Add: Get the common denominator.
Factor first if possible:
Multiply and Divide: Factor First

Rational Inequalities

$\frac{x^2-2x-15}{x-2} \geq 0$ The critical values from factoring the numerator are -3, 5. The denominator is zero at $x=2$.
Place on number line, and test sections.



Sequences
Arithmetic: $a_n = a_1 + (n-1)d$

$$S_n = \frac{n(a_1+a_n)}{2}$$

Geometric: $a_n = a_1\cdot r^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Recursive: Example:
$a_1 = 4; \quad a_n = 2a_{n-1}$

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

Solving Rational Equations:
Get rid of the denominators by mult. all terms by common denominator.

$$\frac{22}{2x^2-9x-5} - \frac{3}{2x+1} = \frac{2}{x-5}$$

multiply all by $2x^2-9x-5$ *and get*

$22-3(x-5) = 2(2x+1)$

$22-3x+15 = 4x+2$

$37-3x = 4x+2$

$35 = 7x$

$5 = x$

Great! But the only problem is that $x=5$ does not CHECK!!!! There is no solution. Extraneous root.
Motto: Always CHECK ANSWERS.

Equations of Circles: $x^2+y^2=r^2$ center origin
$(x-h)^2+(y-k)^2=r^2$ center at (h,k)
$x^2+y^2+Cx+Dy+E=0$ general form

Complex Fractions:
Remember that the fraction bar means divide:
Method 1: Get common denominator top and bottom

$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{\frac{2-4x}{x^2}}{\frac{4x-2}{x^2}} = \frac{2-4x}{x^2}\div\frac{4x-2}{x^2} = \frac{\cancel{2-4x}^{-1}}{\cancel{x^2}}\cdot\frac{\cancel{x^2}}{\cancel{4x-2}} = -1$$

Method 2: Mult. all terms by common denominator for all.

$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{x^2\cdot\frac{2}{x^2}-x^2\cdot\frac{4}{x}}{x^2\cdot\frac{4}{x}-x^2\cdot\frac{2}{x^2}} = \frac{2-4x}{4x-2} = -1$$

Perimeter: add the distances around the outside.

Circumference: $C = 2\pi r = \pi d$

Pythagorean Theorem:
Right Triangles only. $c^2 = a^2 + b^2$
Triples: 3, 4, 5
5, 12, 13
8, 15, 17
7, 24, 25

Trig: Right triangles only
$\sin \measuredangle A = \frac{o}{h}$; $\cos \measuredangle A = \frac{a}{h}$; $\tan \measuredangle A = \frac{o}{a}$
Angle of elevation: from horizontal line of sight up.
Angle of depression: from horizontal line of sight down.

Area:

$A_{triangle} = \frac{1}{2}bh$

$A_{equilateral\ triangle} = \frac{s^2\sqrt{3}}{4}$

$A_{rectangle} = bh$

$A_{square} = bh = s^2$

$A_{parallelogram} = bh$

$A_{rhombus} = bh = \frac{d_1 \cdot d_2}{2}$

$A_{trapezoid} = \frac{1}{2}h(b_1 + b_2)$

$A_{circle} = \pi r^2$

$A_{sector\ of\ circle} = \frac{n}{360}\pi r^2$

$A_{semicircle} = \frac{1}{2}\pi r^2$

$A_{quarter\ circle} = \frac{1}{4}\pi r^2$

Volume and Surface Area:

$V_{rectangular\ solid} = l \cdot w \cdot h$

$SA_{rectangular\ solid} = 2lh + 2hw + 2lw$

$V_{cylinder} = \pi r^2 h$

$SA_{closed\ cylinder} = 2\pi rh + 2\pi r^2$

Error in Measurement:
Relative error = $\frac{|\text{measure-actual}|}{\text{actual}}$
% of Error = Relative • 100%

Permutations:
Arrangement in specific order.
${}_nP_r = \frac{n!}{(n-r)!}$

Data:
5 Statistical Summary: minimum, maximum, median, 1st quartile, 3rd quartile
Quartiles divide data into 4 equal parts.
Percentiles divide data into 100 equal parts.
Percentile rank of score $x = \frac{number\ of\ scores\ below\ x}{n} \cdot 100$, where n is the number of scores.
Mean = average.
Mode = most often (may be more than one answer).
Median = middle.
Outliers = values that are far away from the rest of the data.
Median best describes data if outliers exist.
Range = difference between the maximum and minimum values.

Probability: P(A') = 1 – P(A) complement
P(A and B) = P(A)•P(B) independent
P(A and B) = P(A)•P(B/A) dependent
P(A or B) = P(A) + P(B) mutually exclusive
P(A or B) = P(A) + P(B) – P(A and B) not exclusive
P(B/A) = P(A and B)/P(A) conditional probability
P(B/A) means probability of B given A has occurred.

Box and Whisker Plot: 1st and 3rd quartiles are at the ends of the box, median is a vertical line in the box, and the max/min are at the ends of the whiskers.
Helpful in interpreting the distribution of data.

Literal equations:
$a = b + cd$, solve for c.
$a - b = cd$
$\frac{a-b}{d} = c$
Use same strategies as for solving equations.

Sets:
$A \cup B$ Union - all elements in both sets.
$A \cap B$ Intersection - elements where sets overlap.
A' Complement - elements not in the set.
{ } or ∅ means null set.

Exponential Growth and Decay:
Decay: $y = ab^x$ where $a > 0$ and $0 < b < 1$

Growth: $y = ab^x$ where $a > 0$ and $b > 1$

Sample Problems

This list of problems corresponds roughly to the concept list, though not in "point-by-point" order. They are intended to give you a rough idea of what is meant by the concepts above. This list should be completed, or solved as much as possible with specific questions about how to continue, when you come in for class in August.

Note that knowing these problems only demonstrate that you can perform a variety of skills; it doesn't show whether you really understand what you're doing. Thus, one should always ask not just, "How do I do this problem?" but also, "Why does what I am doing make sense?"

Still, this list is a good place to start. Answers will be forthcoming...we're hoping to have this list with answers posted on the website by the beginning of August. Look for it at seaprep.org!

REGARDING CALCULATORS: Most of these problems are intended to be done <u>without</u> a calculator. Exercise your best judgment on when to use one and when not. Remember, the whole point is to see what you know, so getting an answer by sneaking around the problem isn't in your interest.

Manipulative Algebra

Simplify all of the following:

1) $\left(\sqrt{x}+\sqrt{y}\right)^2$

2) $\sqrt{x^2+y^2}+\sqrt{x^2y^4}$

3) $\left(3x^2\right)^2 3x^2\left(x+3\right)^2$

4) $\dfrac{1}{x-1}-\dfrac{3}{x}$

5) $\dfrac{x^2-3x-4}{x^2-4x+4}\div\dfrac{x^2-5x-6}{x^2-4}$

6) $3\left(2-x^2\right)-\left(4-x\right)x^2-x^2$

7) $\sqrt[3]{x^4}\sqrt[5]{x^4}$

8) $\dfrac{a-3}{a^2-2a}-\dfrac{a-4}{a^2-4}$

9) $\dfrac{6n-2}{6n}\div\dfrac{3n-1}{27}$

Solve all of the following:

10) $3(x-4)-2(3-x)=x+2$ 11) $x^2+4x=2x-4$

12) $2x^2-3(x-4)=x-6$ 13) $(x-1)(x-2)(x-3)(x-4)...(x-100)=0$

14) $(x+1)^{3/4}=27$ 15) $(x-2)^{3/5}=10$ 16) $\sqrt[3]{2x-1}=2$

17) $\dfrac{3+2x}{3-2x}=\dfrac{3}{4}$ 18) $\dfrac{3}{b^2-b}-\dfrac{2}{b-1}=1$

Evaluate each of the following:

19) $4^{3/2}$ 20) 3^{-2} 21) $(-27)^{1/3}$ 22) 10^0

Graph the solutions to each of the following:

23) $3(2-x)<4$ and $2(x-1)<10$ 24) $\sqrt[3]{27x^3}>4$ *or* $x>2^{-1}$

Functions

25) What is the formal definition of a function?

26) Does the graph of a circle represent a function?

27) If $f(x)=x^2-2x+2$, (a) find $f(-2)$, (b) Solve $f(x)=1$.

28) If $f(2)=3$, what point do you know is on the graph of f ?

29) If $f(x)=3x+7$ and $g(x)=2x-1$, then find

a) $f(g(3))$ b) $g(f(3))$ c) $f(g(x))$

d) $f^{-1}(4)$ e) $g^{-1}(x)$ f) $(f+g)(5)$

Linear Functions

30) If a line goes through (7, 4) and has a slope of –2, find the equation of the line in (a) point-slope form, and (b) in slope-intercept form.

31) If a company's sales are growing linearly, and the sales were \$30000 in 1990 and \$48400 in 1992, (a) find the slope of the linear function and (b) explain, using units, what the slope means.

32) Sketch a graph of $3x + 4y = 7$, and find the x- and y-intercepts.

33) Find the intersection of $y=2x+3$ *and* $y=-x-7$.

34) Find the intersection of $2x+3y=4$ *and* $4x-6y=5$.

35) Graph the solutions to $y>3x-7$.

36) Graph the solutions to the system $\begin{cases} y \geq 2x+3 \\ 2x-5y<20 \end{cases}$

37) Find the equation of the line perpendicular to $y=\frac{2}{3}x+7$ that goes through the point (1, 2).

Quadratic Functions and Relations

38) What do you know about each of the following graphs?

a) $y=-3x^2+4x-2$

b) $y=0.04(x-3)^2+4$

c) $y=(x-3)(x-5)$

39) Find the x-intercepts of the parabola given by $f(x)=x^2-7x+1$.

40) Find the equation of the parabola with a vertex at (-3, -4) and that has an x-intercept at 10.

41) Sketch a graph of $x^2+(y-3)^2=16$.

42) Find the equation of the circle that has a center of (2, 7) and that passes through (5, 3).

Exponential Functions and Logarithms

(Note: this is a key item from 2nd year algebra, though it isn't seen at all in geometry. This is a topic that all students should refresh in their memory.)

43) If a population starts at 1000 and is growing at a 4% annual rate, how large will it be after 30 months?

44) When will the population described above reach 2000?

45) Sketch a graph of $y=2^x$, $y=100(1.15)^x$, and $y=\left(\frac{1}{2}\right)^x$.

46) Sketch the graph of $y=\log_2 x$, given the graph of $y=2^x$ as a guide.

47) Evaluate: $\log_3 9$, $\log_4 8$, $\log_{17} 1$, and $\log_2 \frac{1}{4}$.

48) Simplify: $\log 3 + \log 12 - 2\log 2$

49) Solve the equation $50 = 17(1.1)^x$. (If you REALLY got these, write a story problem where you have to solve that equation to find the answer!)

50) What can you say about the number *e*?

51) What is the value of $5000 invested at 8% interest for 5 years if it is compounded
a) every month? b) every day? c) continuously?